Simulation: electron cooling of a bunched beam

M. Takanaka

RIKEN (The Institute of Physical and Chemical Research), 2-1 Hirosawa, Wako-shi 351-0198, Japan

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Abstract

In performing particle-tracking simulations, electron cooling of a bunched beam has been studied. Bunch shortening makes the beam space-charge-dominated at low energies. Characteristics of the stable beam under a cure for a transverse instability are described.

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1. Introduction

The Radioisotope (RI) Beam Factory [1] has a plan to have an electron-RI beam collider (e-RI Collider). Electron cooling (EC) of a bunched ion beam at low energies has been studied in order to satisfy ion-beam requirements at the collider, in performing particle-tracking simulations where the three-dimensional (3D) space-charge field is taken into account. Fundamental characteristics of the stable bunched beam under a cure for a transverse instability are described in this paper, while instabilities during the EC of a bunched beam and the cures have been described as well as the details of the simulation method [2].

2. Simulation method

In the simulation, the following sources of forces acting on particles have been taken into account:

- 3D space-charge field induced by a beam itself.
- Transverse dipole broadband impedance.
- EC force [3].
- Solenoid field and toroid field of the EC section.
- RF field for beam bunching.
- A uniform focusing section and a drift EC section to which the ring lattice is simplified.

The transverse field is calculated using the charge distribution on transverse 101 × 101 cells across the cross-section of 80 mm² of which only the central cell consists of 13 × 13 cells in it. Since the simulation has shown that the cooled beam is extremely centralized transversely even when the beam becomes space-charge dominated, such a
twofold cell structure has been used eventually. Seven field maps are made for seven divisions along the bunch, the structure being formed at each division. The field at any particle is estimated making the field interpolation between two neighboring maps and taking account of the charge-line-density distribution. The field calculation contains contributions from image charges reflected from the perfectly-conducting smooth wall of the vacuum chamber [4].

The transverse charge distribution is assumed to be Gaussian with a standard deviation \( \sigma \) which is much smaller than the radius of the vacuum chamber \( b \). Then, the longitudinal monopole, dipole, and quadrupole space-charge impedances [5] are

\[
\frac{Z_{n}^{0}}{n} = \frac{1}{2} \sum_{m=0}^{\infty} \frac{Z_{n}^{m}}{n} \frac{dI^{m}}{ds}
\]

where \( r \) is the radial position, \( \beta \) and \( \gamma \) the relativistic constants of the beam velocity \( v \), \( Z_{0} \) impedance of the free space, \( n = \omega_{0}/\omega_{0} \), \( \omega_{0} \) being the revolution frequency, \( \sigma^{2} = (\sigma_{x}^{2} + \sigma_{y}^{2})/2 \), and \( \text{erf}(x) \) the error function.

The longitudinal field is described approximately as

\[
g_{0}(r) \approx -\int_{r}^{\infty} \frac{e^{-x}}{x} \, dx + 2 \log \left( \frac{b}{r} \right)
\]

\[
g_{1}(r) \approx \frac{2r}{\sigma^{2}} \left[ \text{erf} \left( \frac{r}{\sqrt{2}\sigma} \right) - \frac{2}{\sigma} \sqrt{\frac{2}{\pi}} \exp \left( -\frac{r^{2}}{2\sigma^{2}} \right) \right]
\]

\[
g_{2}(r) \approx -\left( \frac{1}{2\sigma^{2}} + \frac{1}{r^{2}} \right) \exp \left( -\frac{r^{2}}{2\sigma^{2}} \right) + \frac{1}{r^{2}}
\]

\[
+ \frac{r^{2}}{4\sigma^{4}} \int_{x=r}^{\infty} \frac{e^{-x}}{x} \, dx - \frac{r^{2}}{b^{2}}
\]

where \( r \) is the radial position, \( \beta \) and \( \gamma \) the relativistic constants of the beam velocity \( v \), \( Z_{0} \) impedance of the free space, \( n = \omega_{0}/\omega_{0} \), \( \omega_{0} \) being the revolution frequency, \( \sigma^{2} = (\sigma_{x}^{2} + \sigma_{y}^{2})/2 \), and \( \text{erf}(x) \) the error function.

It is said [6] that the space-charge impedance attenuates to become half around 14–19 GHz for the beam size 3–4 mm in the current simulation condition. Thus, the impedances are assumed to attenuate exponentially beyond \( 10\omega_{c} \approx 12 \text{ GHz} \):

\[
\exp \left( 1 - \frac{\Omega}{10\omega_{c}} \right)
\]

where \( \omega_{c} = c/b \) is the characteristic resonance frequency for the broadband impedance model, \( c \) being the light velocity.

A parabolic-bunch length of a space-charge dominated beam \( l \) is dependent on RF voltage \( V_{RF} \) and the harmonics \( h \):

\[
l \propto \frac{1}{hV_{RF}}.
\]

An RF system supplies fundamental RF voltage and 3rd-harmonic RF voltage for bunching of a coasting beam and a bunched beam shorter than one-third bunch spacing, respectively. The two cavities installed at an azimuth have to be located at the azimuth \( \pi \) from the EC section. Then, EC works most effectively, because in this location the momentum spread at the EC section is the smallest along the ring.

Since a single bunch phenomenon during the EC is an exclusive interest, every bunch is assumed to behave the same. Using 40,000 macro-particles, the EC process has been simulated for a \( ^{238}\text{U}^{92+} \) beam of 4 mA, or \( 5.4 \times 10^{6} \) ions per bunch at 150 MeV/u.

3. Bunched beam in equilibrium

A coasting beam pre-cooled in Accumulator Cooler Ring is injected into the e-RI Collider where the major resonance \( Q_{x} + Q_{y} = 8 \) (the nearest to and below the operation point) due to the toroid field is controlled by using two skew quadrupoles in order to suppress the transverse instability. The simulation results of the EC are shown in Fig. 1. The beam reaches equilibrium among EC, the space-charge effects, the RF-field effect, and perhaps the resonance effect due to the imperfection of the resonance control around 160 ms.
Fundamental characteristics of the bunched beam in equilibrium are shown hereafter using the simulation results at 160 ms.

3.1. Longitudinal characteristics

It is one of the space-charge effects during a revolution, as shown in Fig. 2, that a durable energy gain from the longitudinal space-charge field is nearly compensated by a momentary energy gain from the RF field. Just before and just after the RF section the longitudinal distribution is one with the opposite slope in the phase space. The bunch length becomes longest at the RF section and shortest at the EC section. The longitudinal motion of the beam is not synchrotron oscillation, but the up-and-down movement at the tune of $Q = 1$. The tune is dependent on components of an RF system and their location relative to an EC section.

The bunching factor is 27, being defined as the ratio of the bunch spacing to the bunch length. Some excess over the parabolic profile is seen around the bunch center.

The bunch profile has a frequency spectrum $A^0(\Omega, k)$ shown in Fig. 3 which is the result of Fourier transformation of the monopole moment data $\lambda(t, z)$ sampled every one-eighth revolution.
for 100 revolutions

\[ \lambda(t, z) = \sum_{\Omega, k} A^0(\Omega, k) \times \exp \left( -i(\Omega - kh) t + khz \right) \]

where \( z \) is the position from the synchronous particle. The spectrum has the following features:

- Along \( \Omega/\omega_0 = kh \) signals due to the parabolic profile.
- Along \( \Omega/\omega_0 = kh + i \) \((i = \) non-zero integer\) signals due to the up-and-down movement.
- Along the two-spread-wing frame reflection signals from the longitudinal space-charge impedance. The position of the signals is described by the following equation derived by applying the Vlasov equation to the case of a bunched beam with little momentum spread:

\[ \Omega - n\omega_0 = \pm n\omega_0 \sqrt{\frac{iq_\eta I_B}{2\pi\beta^2 E} \left[ Z_{ij,n}^{\text{eff}} \right]_{sp}} \]

where \( I_B \) is the instantaneous beam current averaged over the bunch, and \( Z_{ij,n}^{\text{eff}} \) the longitudinal effective space-charge impedance. The dynamics of the signal production is equal to that for double-peaked Schottky spectra of cooled coasting beams. Since the longitudinal impedance has been assumed to attenuate
beyond $k = 234$, or 12 GHz, the signal position deviates inward from on the straight lines beyond $k = 234$.

As shown in Fig. 4, the energy gain of an ion from the longitudinal space-charge field and the RF field per revolution has been checked to be dependent on the betatron amplitude. The attenuation of the longitudinal space-charge impedance beyond 12 GHz makes the distortion smooth around the bunch edges ($|z| = 5.5$ cm). It can be deduced from Fig. 4 that ions with large (or small) amplitudes distribute towards (or far from) the bunch center, the longitudinal profile being kept approximately parabolic, as the momentum spread becomes small.

Fig. 6. Incoherent tune distributions. In the lowest the curve for $5Q_x$ is scaled down to $\frac{1}{4}$ vertically.
3.2. Transverse characteristics

The transverse charge distribution of the bunch at the EC section is shown in Fig. 5. The beam has a large width at the bunch center, as deduced above. The charge spatial density is considerably low at the bunch center.

3.3. Incoherent tunes

Fig. 6 shows incoherent tune distributions. Ninety percent of all the ions are below the major $Q_x + Q_y = 8$ resonance line and are in the head and the tail region ($|z| > \sim 5$ mm). The rest of the ions are in the central region ($|z| < \sim 5$ mm).

The nearer to the bunch ends, the larger is the amplitude of the horizontal head-tail-mode-1 coherent oscillation induced through the momentum dispersion function by the longitudinal up-and-down movement. Ion’s betatron oscillation is mixed with the coherent oscillation. This is why the horizontal tune shift in the head and the tail region is smaller than the vertical shift.

Although the bare incoherent synchrotron tune is 0.15 for the 3rd-harmonic RF voltage 0.78 MV, most of the incoherent tunes are nearly zero except for those for ions around the bunch center. This means that the beam synchrotron oscillates hardly because of the RF-wave-form distortion.

4. Conclusions

It has been shown using results of the particles-tracking simulation where the 3D space-charge field is calculated that stable electron cooling of a bunch beam is carried out up to the 3rd RF harmonic voltage of 0.78 MV under the cure for transverse instability. Characteristics of the bunched beam in equilibrium have been shown: the up-and-down movement in the longitudinal phase space, the RF-wave-form distortion, the excess over the parabolic profile, the transverse profile with a large width and a low density at bunch center, and large incoherent tune shifts.

References