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Numerical simulation of crystalline ion beams in storage ring

I. Meshkov[a], D. Möhl[b], T. Katayama[c,*], A. Sidorin[a], A. Smirnov[a], E. Syresin[a], G. Trubnikov[a], H. Tsutsui[c]

[a] *Joint Institute for Nuclear Research (JINR), Joliot Curie 6, Dubna 141980, Russia*
[b] *European Organisation for Nuclear Research (CERN), CH 1211 Geneva, Switzerland*
[c] *Institute of Physical and Chemical Research (RIKEN), CNS, The University of Tokyo, Hirosawa 2-1, Wako, Saitama 351-0198, Japan*

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Abstract

The use of crystalline ion beams can increase luminosity in the collider and in experiments with targets for investigation of rare radioactive isotopes. The ordered state of circulating ion beams was observed at several storage rings: NAP-M (Proceedings of the Fourth All Union Conference on Charged Particle Accelerators, Vol. 2, Nauka, Moscow, 1975 (in Russian); Part. Accel. 7 (1976) 197; At. Energy 40 (1976) 49; Preprint CERN/PS/AA 79-41, Geneva, 1979) (Novosibirsk), ESR (Phys. Rev. Lett. 77 (1996) 3803) and SIS (Proceedings of EPAC'2000, 2000) (Darmstadt), CRYRING (Proceedings of PAC'2001, 2001) (Stockholm) and PALLAS (Proceedings of the Conference on Applications of Accelerators in Research and Industry, AIP Conference Proceedings, p. 576, in preparation) (München). New criteria of the beam orderliness are derived and verified with a new program code. Molecular dynamics technique is inserted in BETACOOL program (Proceedings of Beam Cooling and Related Topics, Bad Honnef, Germany, 2001) and used for numerical simulation of crystalline beams. The sudden reduction of momentum spread in the ESR experiment is described with this code. The simulation shows a good agreement with the experimental results. The code has then been used to calculate characteristics of the ordered state of ion beams for the MUSES Ion Ring (IR) (MUSES Conceptual Design Report, RIKEN, Japan, 2001) in collider mode. A new strategy of the cooling process is proposed which permits to increase significantly the linear density of the ordered ion beam and thereby the luminosity of electron–ion colliding experiments.

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1. Criteria of beam ordering

The common criterion of the "beam crystallisation" is a decrease of the particle temperature T below interparticle potential energy U. When one deals with an ion beam circulating in a ring [1–7],

*Corresponding author.
E-mail address: katayama@postman.riken.go.jp (T. Katayama).

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one has to consider the particle interaction in "*the particle rest frame*" (PRF). Then the crystallisation criterion can be written as follows:

$$\Gamma = \frac{U}{T} = \frac{Z^2e^2}{aT} > 1. \tag{1}$$

Here Ze is the ion charge and a the interparticle distance (in PRF). Criterion (1) does fit well in the case of a homogeneous crystal. However, if the ion beam is cooled (by electron or/and laser cooling), the ion parameters differ significantly in longitudinal and transverse dimensions. As a rule, ion transverse temperature is much larger than the longitudinal one,

$$T_\perp \gg T_\parallel. \tag{2}$$

The relation between transverse beam size $\sigma_\perp$ and the particle distance along the ring circumference $a_\parallel$ depends strongly on the beam state—its intensity and the ion temperature. Particularly, parameters $\sigma_\perp$ and $T_\perp$ are connected when ions circulate in a ring. In PRF we have

$$T_\perp = Mc^2\beta^2\gamma^2\left(\frac{2\pi Q_{\rm bet}}{C_{\rm ring}}\sigma_\perp\right)^2, \quad V_\perp = c\beta\gamma\frac{2\pi Q_{\rm bet}}{C_{\rm ring}}\sigma_\perp \tag{3}$$

where $M = Am_{\rm p}$ is the ion mass, $m_{\rm p}$ the proton mass, A the atomic mass number, β, γ the ion Lorentz factors, $Q_{\rm bet}$ the betatron tunes, $C_{\rm ring}$ the ring circumference, and $V_\perp$ the ion average transverse velocity in PRF.

The ion longitudinal temperature and average distance between ions in a coasting beam (in PRF) can be written as follows:

$$T_\parallel = M_{\rm i}c^2\beta^2(\Delta p/p)^2, \quad a_\parallel = \frac{\gamma C_{\rm ring}}{N_{\rm i}} \tag{4}$$

where $\Delta p/p$ is the ion momentum spread in the laboratory reference frame (LRF) and $N_{\rm i}$ is the ion number in the beam. Ion collisions occur mostly in transverse plane and impact parameter ρ is related to the scattering angle θ in accordance with well-known Rutherford formula:

$$\rho_{\min} = \frac{Z^2e^2}{2T_\perp}{\rm ctg}\frac{\theta}{2}. \tag{5}$$

Now we can formulate *the first criterion of the beam ordering*: the scattering angle has to be so large that the longitudinal component of the scattered ion changes its sign and the ion moves backwards keeping (in average) its position in the chain. It means that $\theta V_\perp > V_\parallel$ and, as a result, one requires

$$\rho_{\min} \sim \frac{Z^2e^2}{T_\perp}\frac{1}{\theta} \approx \frac{Z^2e^2}{MV_\perp V_\parallel}. \tag{6}$$

Substituting this expression for $\rho_{\min}$ in an evident inequality $\rho_{\min} < a_\parallel$, one can write *the first criterion of the beam ordering* in the following form:

$$\Gamma_1 = \frac{Z^2e^2}{a_\parallel\sqrt{T_\parallel T_\perp}} < 1. \tag{7}$$

The second criterion of the beam ordering can be called "the nonmissing condition", that means that ions do not pass each other without an appropriate scattering by an angle θ. This requirement is fulfilled if time interval of the ion travelling along the distance equal to $\rho_{\min}$ (6) exceeds at least a quarter of betatron oscillation period in PRF. This condition gives us

$$V_\parallel < \frac{2\rho_{\min}}{C_{\rm Ring}} Q_{\rm bet}\gamma\beta c. \tag{8}$$

Substitution of $V_\perp$ from Eq. (3) to Eq. (6) and the obtained expression for $\rho_{\min}$ in (8) allows us to formulate *the second criterion of the ordering*:

$$\Gamma_2 = \frac{Z^2e^2}{T_\parallel\sigma_\perp} > \pi. \tag{9}$$

There are four rings up to now, where the ion beam ordering was observed ([8, Table 1]). The value of plasma parameter Γ_2 does not satisfy the criterion (9) for light ions $^{12}C^{6+}$ and $^{20}Ne^{10+}$ ([8, Table 1, ESR experiments]). This fact has good agreement with experiments where the reduction of momentum spread was not observed for light ions in ESR experiments.

2. Experimental studies of ordered beams

The experiments with the cooled ion beams performed on ESR, SIS and CRYRING storage rings have allowed to study the beam-ordered state features. On ESR, the momentum spread of a uranium beam at 360 MeV/u cooled by 0.25 A electron beam drops to around 1000 stored ions from a value of $\Delta p/p = 5 \times 10^{-6}$ to 5×10^{-7} corresponding to a change of the longitudinal temperature by two orders of magnitude (Fig. 1) [9].

After beam ordering, the momentum spread stays on a constant level of $\Delta p/p \sim 5 \times 10^{-7}$ and this value corresponds to the revolution frequency variation by the instability of the bending magnet power supplies. Experiments with short averaging times have evidenced that the actual momentum spread is smaller, typically $\Delta p/p \sim 2 \times 10^{-7}$ [9].

Intrabeam scattering heating force (IBS) has well-known distribution for the large number of particles and goes to infinity with the decreasing of 6D phase space of ion beam for a large number of particles and temperature. Then the particle number and momentum spread are decreased with time (Fig. 1); the ion beam goes to the ordered state when the crystallisation conditions are satisfied. The dependence of IBS on the ion temperature is changed to the bell shape (Fig. 2) [10]. Last equilibrium point between IBS and electron cooling forces (ECOOL) (Fig. 2A) corresponds to the ion temperature before the sudden reduction of the momentum spread as shown in Fig. 1. In this situation, the ion temperature jumps to the equilibrium point between ECOOL and other heating processes (Fig. 2B), which can be defined by the stability of magnetic fields and power supplies, scattering on residual gas atoms, etc.

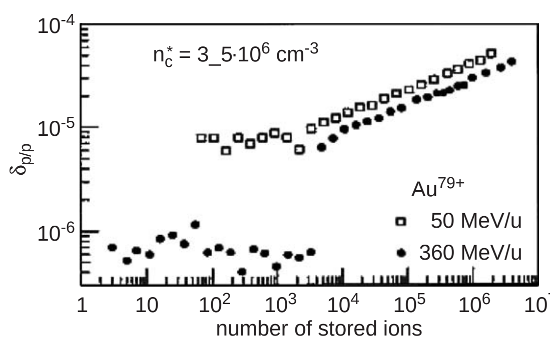

Fig. 1. Momentum spread for different power supply stability of electron cooler. The power supply for energy of 360 MeV/u is more stable than for energy 50 MeV/u (ESR experiment).

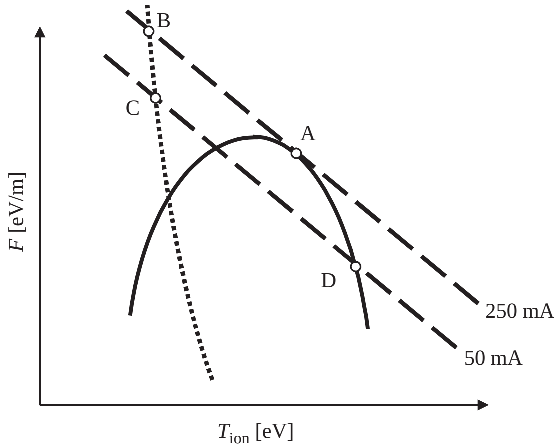

Fig. 2. The dependence of cooling and heating forces on the ion temperature. Solid line corresponds to IBS, dashed line—to ECOOL with negative sign, dotted line—to other heating process.

One can explain some experimental results from this idea. In the case of large additional heating from the electron cooler power supply, the sudden reduction of the momentum spread was not observed (Fig. 1). This means that equilibrium between ECOOL and heating from the power supply was achieved before the disappearance of equilibrium between IBS and ECOOL.

Let us formulate the following conditions when the sudden reduction of momentum spread can be observed:

- Ordered state criteria have to be met (lower limit of ion number).
- IBS heating force can become smaller than electron cooling one (upper limit of ion number).
- The additional heating forces should be smaller than IBS before the momentum reduction.

As follows from Fig. 2, the hysteresis of momentum spread jumps can be checked. No way to increase the ion number during the experiment. This phenomenon can be verified experimentally by the variation of the electron beam current when the sudden reduction of momentum spread appears. For example, when the momentum reduction is achieved for electron beam current 250 mA (Fig. 2A and B), one can decrease the electron current to the value 50 mA (Fig. 2B and C). In this case we can find hysteresis point (Fig. 2C) opposite to the equilibrium point for the electron beam current 50 mA (Fig. 2D).

3. Numerical simulation of the ordered state

The behaviour of the ion beam parameter evolution can be explained using 3D diagrams of the beam parameter growth rates due to the intrabeam scattering [11]. IBS growth rates as functions of the beam emittance and momentum spread were calculated in accordance with the generalised Piwinski model.

The condition $\Gamma_2 = \pi$ describes the following relation between beam emittance and momentum spread: $\varepsilon \sim (\Delta p/p)^{-4}$. In the twice-logarithmic scale, this dependence is presented in Fig. 3 as a black solid line. Experimental points of the equilibrium beam parameters lie over and right of this threshold line when the particle number is larger than the value corresponding to the sudden reduction of momentum spread.

The numerical simulation using MD method is presented in Fig. 3. In ESR experiments, the magnitudes of beam emittances and momentum spread before the momentum reduction are defined by equilibrium between IBS growth and cooling rates (series of points in Fig. 3a) [12]. The experimental points lie on the line, which approximately corresponds to equal values of longitudinal and transverse growth rates. The particle number decreases with time and when Γ_2 parameter exceeds π, the cooling forces suppress the IBS ones and the beam comes to the ordered state (last experimental point in Fig. 3a).

The results of calculation using MD technique (Fig. 3) are in good qualitative agreement with 3D diagram [11]. All the calculations were performed for the same particle number—5×10^5 at different values of the cooling strength and different initial conditions. At the first stage of the beam cooling, all the lines have the same angular inclination determined by ratio between cooling rates in transverse and longitudinal degrees of freedom. In the case of uniform cooling $\varepsilon \sim (\Delta p/p)^2$, one can see this dependence in the initial part of the beam phase trajectory independently on initial point. It means that IBS process does not affect the cooling process at the initial phase space of the beam.

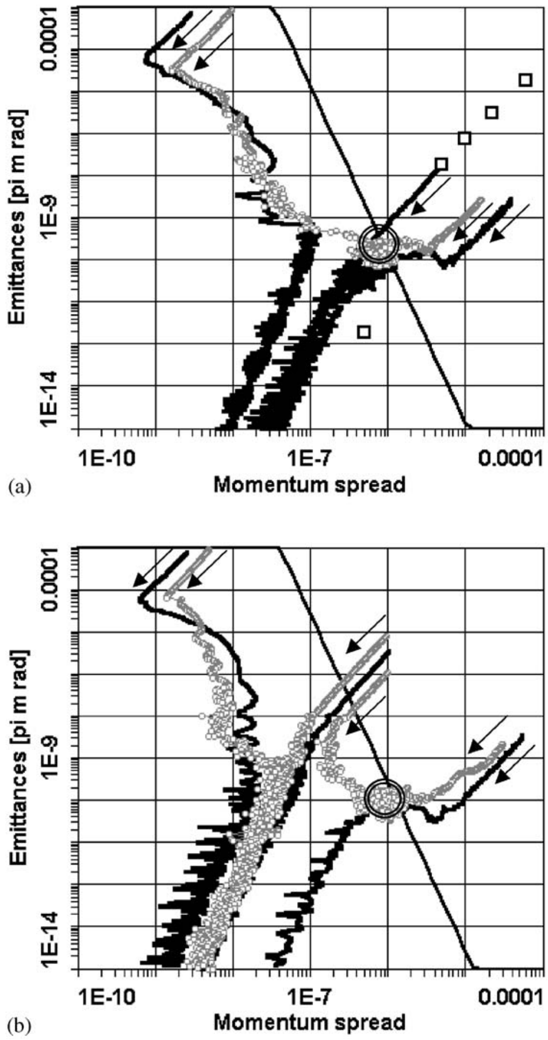

Fig. 3. Evolution of the ion beam parameters during the cooling process for different initial emittances (momentum spread) and cooling strength: (a) ESR parameters; (b) MUSES project of ion ring. Solid black line corresponds to $\eta = 2 \times 10^{-2}$, grey circles to $\eta = 5 \times 10^{-3}$, straight line is criterion $\Gamma_2 = \pi$, and square points to ESR experiment. $^{197}Au^{79} + 360\,MeV/u$, $N = 5 \times 10^5$. Double circle means equilibrium point, when ordered state is not achievable.

The ordered state MD calculations [11] are in good agreement with IBS growth rates predicted before by Piwinski model. The IBS growth rates in the ordered state calculated using MD method are substantially less than those predicted by Piwinski model and at the cooling strength $\eta = 2 \times 10^{-2}$ (cooling rates $\Omega_{\rm cool} = \eta / T_{\rm rev} = 4 \times 10^4$ Hz) the beam emittance and momentum spread at the particle number $N = 5 \times 10^5$ decrease to very small values.

4. MUSES ion ring

The project of MUSES electron–ion collider [7] has the goal to determine the charge distributions of neutron- and proton-rich radioactive nuclei where exotic structures such as neutron halo and skin are predicted. Challenges are related to collider high luminosity and to large acceptance and high-resolution electron spectrometer.

The achievement of ordered ion beams could increase the luminosity in colliding experiments by several orders of magnitude [13]. But one might conclude from the ESR experiments that for existing cooling systems, the ordered state can exist only for a small number of particles $\sim 10^3$. In this case the ordered beam with small momentum spread and small transverse size has no large advantage in comparison to ordinary ion beams with a large number of particles. To resolve this problem, a new strategy of the cooling process to achieve the ordered state with a large number of particles, up to 10^6, is proposed.

This idea is illustrated by the numerical simulations of the beam dynamics using MD technique for MUSES ion ring. Unlike the ESR case, the final state of the beam parameters depends on initial values of emittance and momentum spread. At large initial emittance and small momentum spread, the ion beam can achieve the ordered state for a smaller cooling strength (Fig. 3b, $\eta = 5 \times 10^{-3}$, $\Omega_{\rm cool} = 2 \times 10^4$ Hz). The search for a criterion for the optimal lattice structures is the main task of the further investigations.

To achieve ordered ion beams with a large number of particles and with a realistic cooling force, a special strategy of the cooling process should be elaborated. When the ion beam stays in equilibrium between IBS and cooling we may apply additional heating in the transverse direction. For example, heating by an RF-kicker can be used. Initially, the momentum spread and emittances will increase together. When the beam parameters have come around the maximum of the IBS longitudinal component, the momentum spread will decrease once again. After that we can switch off the additional heating and the ion beam will continue to cool down to the ordered state.

The same idea can be proposed for the ESR ring if another lattice setting can be found when the final state of cooled ion beam depends on initial values of momentum spread and emittances. The experimental verification of the new strategy to achieve the ordered ion beam with large density can open new possibilities in the accelerator physics.

5. Conclusion

MD technique is inserted in BETACOOL code for simulation of crystalline beams. New criteria of the beam orderliness are derived. They are shown to be in good agreement with MD simulation results. Comparison with the results of other theoretical models of intrabeam scattering (IBS) gives further insight in the physics of beam ordering. Parameters of the ion beam before the momentum reduction are defined by the equilibrium between cooling force and IBS. When the cooling rate exceeds the maximum of the IBS rate, the ion beam can reach to the ordered state. For typical parameters of existing storage rings and realistic electron cooling systems, the maximum number of particles when the ordered state can be achieved is about $10^3 - 10^4$. The new strategy, which allows us to increase the particle number up to 10^6 in an ordered state of an ion beam, is proposed.

Acknowledgements

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