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Physical model of the crystalline beam in a storage ring[☆]

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Abstract

Simple physical model of two-dimensional state of a crystalline ion beam in a cooler storage ring is presented. The analysis based on ion–ion collisions gives the criterion of such a state existence and an estimate of ion oscillation frequency at this state. The criterion agrees with one derived earlier by Hasse for an ion crystalline beam at zero temperature using Wigner–Seitz crystal theory.

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1. Introduction

Crystalline (ordered) beam state since first experimental discovery of phase transition of an ion beam in a storage ring [1] and its explanation [2] became an attractive object of intense experimental and theoretical studies. Theory of three-dimensional crystalline beam developed later [3,4] predicts certain characteristics of such a beam state. Nevertheless, the theoretical approach presented in Refs. [3,4] is rather formal, based on Hamiltonian formalism. Computer simulations, which intimate strongly an existence of crystalline beam, do not give a proper physical explanation of the phenomenon. This paper is an attempt to overcome (at least, partially) this imperfection.

2. Model of two-dimensional crystalline beam

In “the multilayer crystalline beam”, ions oscillate in the transverse planes around some equilibrium positions (trajectories) with small amplitudes (Fig. 1). The equilibrium trajectories are parallel to that one in the case of a “conventional” beam, i.e. the axis of magnetic focusing system.

If the beam is cooled “very deeply” we have the so-called “particle rest frame” (PRF), which moves with average ion velocity, an approximate equality of ion velocity and temperature components

$$V_{\perp} \sim V_{\parallel} \equiv V, \quad T_{\perp} \sim T_{\parallel} \equiv T. \quad (1)$$

Ion–ion collisions in PRF occur with the average periodicity

$$\tau_{\text{coll}} \sim \frac{l_{\parallel}}{V_{\parallel}}, \quad l_{\parallel} \sim \frac{\gamma C_{\text{Ring}}}{N_i}. \quad (2)$$

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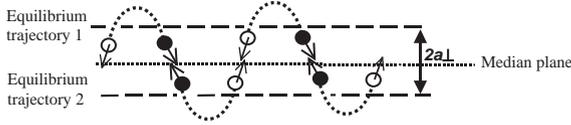


Fig. 1. The ion trajectories in the particle rest frame when zig-zag structure is formed: ions oscillate between two (left ○ and right ●) collision points.

Here l_{\parallel} is interparticle distance in the ring of the circumference C_{Ring} , N_i is the ion number in the beam. Momentum transfer (velocity change) in a collision is of the order of

$$\Delta V_{\perp} \sim \theta V, \quad \theta \sim \frac{Z^2 e^2}{T \rho}, \quad \rho = a_{\perp} + x \quad (3)$$

where θ is scattering angle, Ze and T is ion charge and temperature, ρ is the impact parameter, which has a value of the order of the ion beam average transverse size $2a_{\perp}$, x is the instant value of the ion displacement from equilibrium trajectory. Then one can estimate the transverse repulsion force

$$F_{\perp} = \frac{\Delta p_{\perp}}{\tau_{\text{coll}}} = \frac{M \Delta V_{\perp}}{\tau_{\text{coll}}} \sim \frac{Z^2 e^2}{l_{\parallel}(a_{\perp} + x)} \quad \text{and} \quad \bar{F}_{\perp} = \frac{Z^2 e^2}{l_{\parallel} a_{\perp}}. \quad (4)$$

In Laboratory Reference Frame (LRF) we have the equation of betatron oscillations

$$\gamma M \left(\frac{d^2 x}{dt^2} + Q_{\text{bet}}^2 \omega_0^2 (a_{\perp} + x) \right) = \frac{F_{\perp}}{\gamma}, \quad \omega_0 = 2\pi \frac{\beta c}{C_{\text{Ring}}} \quad (5)$$

where β , γ are the ion Lorentz factors, M is the ion mass, Q_{bet} and ω_0 are betatron tune and ion revolution frequency in the ring of the circumference C_{Ring} , c is the speed of light. Linearization of Eq. (5) gives us the condition when the string becomes unstable and zig-zag state formation begins

$$\gamma M Q_{\text{bet}}^2 \omega_0^2 a_{\perp} \leq \frac{\bar{F}_{\perp}}{\gamma}. \quad (6)$$

At the zig-zag state the ion oscillation frequency (betatron tune) is shifted by ΔQ value

$$Q_{\text{bet}} \rightarrow Q_{\text{bet}} + \Delta Q, \quad 2Q_{\text{bet}} \Delta Q = \frac{\bar{F}_{\perp}}{\gamma^2 M a_{\perp}^2 \omega_0^2}. \quad (7)$$

Substituting in Formulae (6) and (7) the expression for \bar{F}_{\perp} from Formula (4) and l_{\parallel} from Formula (2) we find *The First Criterion* of 2D state formation

$$\Gamma_{2D} = \frac{Z^2}{A} \frac{r_p N_i}{4\pi^2 Q_{\text{bet}}^2 \beta^2 \gamma^3} \frac{C_{\text{Ring}}}{a_{\perp}^2} \geq 1. \quad (8)$$

and betatron tune shift value

$$\Delta Q = \frac{\Gamma_{2D}}{2Q_{\text{bet}}} \sim \frac{1}{2Q_{\text{bet}}}. \quad (9)$$

The beam structure of zig-zag form assumes an additional condition for ion beam parameters, which follows from the model presented in Fig. 1: collision time has to be less than one half of betatron oscillation period in PRF

$$\tau_{\text{coll}} \leq \frac{T'_{\text{bet}}}{2}.$$

It gives us *The Second Criterion* of 2D state formation

$$N_i \geq \frac{\gamma^2 \beta c Q_{\text{bet}}}{V_{\parallel}}. \quad (10)$$

For typical parameters of ESR experiments [5]

$$^{238}\text{U}^{92+}, \quad 360 \text{ MeV/u}, \quad a_{\perp} \sim 3 \mu\text{m}, \quad V_{\parallel} \sim 100 \text{ m/s}.$$

Then we find from *The First Criterion* $(N_i)_1 > 1.6 \times 10^6$ and from *The Second Criterion* $(N_i)_2 > 8 \times 10^6$. For NAP-M experiment [1]

$$\text{protons}, \quad 65 \text{ MeV}, \quad a_{\perp} \sim 0.1 \text{ mm}, \quad V_{\parallel} \sim 130 \text{ m/s}$$

the same estimates give us $(N_i)_1 > 1.1 \times 10^9$ and $(N_i)_2 > 1.3 \times 10^6$.

Thus, both experiments did not meet the requirements formulated above.

3. The Hasse Criterion

The critical parameter introduced in Ref. [6] is

$$\lambda = \frac{a_{\text{WS}}}{d} \quad (11)$$

where a_{WS} is so called Wigner–Seitz radius

$$a_{\text{WS}} = \left(\frac{3}{2} \frac{Z^2 r_p C_{\text{Ring}}^2}{A \beta^2} \right)^{1/3}$$

d is the interparticle distance in the crystal. It allows to formulate The Crystal State Criterion at $T = 0(!)$ [6]. In accordance with it the transition between string and zig-zag structures takes a place when λ value exceeds 0.709.

In an ion beam we have

$$d = \left(\frac{\pi a_{\perp}^2 C_{\text{Ring}}}{N_i} \right)^{1/3}.$$

Then the critical parameter λ (Formula 11) can be written as

$$\lambda = \frac{1}{\pi} \left(\frac{3}{8} \frac{Z^2 N_i r_p C_{\text{Ring}}}{A \beta^2 a_{\perp}^2} \right)^{1/3}.$$

We see that λ and $\Gamma_{2\text{D}}$ are connected by the expression

$$\lambda = \left(\frac{3}{2\pi} Q_{\text{bet}}^2 \right)^{1/3} \Gamma_{2\text{D}}^{1/3} \approx \Gamma_{2\text{D}}^{1/3}.$$

Thus, $\Gamma_{2\text{D}} \geq 1$ Criterion agrees in general with the Hasse Criterion.

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