Simulation of electron cooling process in storage rings using BETACOOL program

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ABSTRACT

An electron cooling method is widely used for ion beam parameter control in the storage rings. Presently there are about 20 storage rings in operation and under construction, which are equipped with electron cooling devices. The BETACOOL program elaborated for simulation of electron cooling process is actively used in several research centres: CERN, Juelich-FZ, DESY, RIKEN, Fermilab. This report discusses new development of the BETACOOL program aiming to include into computation algorithm the following processes: intrabeam scattering, ion scattering on residual gas atoms, interaction of the ion beam with internal target and some others. The BETACOOL is programmed with object oriented method using C++ Builder3 and is based on BOLIDE package (Beam Optic Library & Interface Development Environment), which is dedicated to fast elaboration of the physics and mathematics applications. The code structure, program interface and some results of calculation are presented.

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1. GENERAL DESCRIPTION OF THE PHYSICAL MODEL

The physical model which is investigated using BETACOOL program [1] is based on the following general assumptions:

1) the ion beam has Gaussian distribution on all degrees of freedom, which does not change during the process.
2) algorithm for analysis of the problem consists in a solution of the equations for root mean square values of the beam phase space volumes of three degrees of freedom.
3) maximum of all distribution function coincide with equilibrium orbit.

The evolution of the ion beam parameters during its motion inside the storage ring is described by the following system of four differential equations:

$$\begin{cases} \dot{N} = N\sum_j \frac{1}{\tau_{life,j}}, \\ \dot{\varepsilon}_i = \varepsilon_i \sum_j \frac{1}{\tau_{i,j}}, \end{cases} \tag{1}$$

where N is particle number, ε_i is mean value of the beam phase space volume (index i corresponds to degree of freedom). For transverse degrees of freedom ε_i is corresponding to the beam emittance, for longitudinal degree of freedom it given by the following expression:

$$\varepsilon_{lon} = \begin{cases} (\frac{\Delta p}{p})^2, & \textit{coasting beam}; \\ (\frac{\Delta p}{p})^2 + \frac{1}{\Omega_s^{\,2}}[\frac{d}{dt}(\frac{\Delta p}{p})]^2, & \textit{bunched beam}. \end{cases} \quad (2)$$

In Eq. (2) upper line corresponds to coasting beam, lower line to bunched beam with constant parameters (for changing synchrotron frequency it is necessary to use adiabatic invariant instead of energy and presently depression of the synchrotron tune due to action of the beam space charge is not taken into account during dynamics simulation). Characteristic times are functions of all three emittances and particle number, life time in the first equation is positive for particle injection and negative for particle losses.

Index *j* in Eq.(1) is the number of process involved into calculations. We present here the program version, where a calculation of the intra beam scattering (IBS), electron cooling and decay of unstable nuclei are realized. However, program structure is designed in such a way that permits to include into calculation any process, which can be described by cooling or heating rates.

During numerical solution of the system (1) the parameters characterizing beam stability are calculated also. There are incoherent betatron tune shift value, depression of the synchrotron tune, dimensionless parameters describing the beam from the side of longitudinal and transverse coherent instabilities [2]. Presented version of the program provides also luminosity calculation in the bunch-bunch collisions for head on and crossing angle modes by numerical evaluation of the overlapping integral for bunches with the Gaussian space charge density distributions. In nearest future we plan to include into the algorithm luminosity calculations in an experiment with internal target.

2. ALGORITHMS OF THE NUMERICAL CALCULATIONS

Numerical solution of the system (1) is performed using Euler method with automatic step variation. The use of high order Runge-Kutt methods meets certain problems related to long time of the equation right parts calculation.

A. Calculations of the electron cooling time rates

Electron cooling time rates are calculated through averaging of the action of friction force inside cooler over phases of betatron and synchrotron oscillations and over Gaussian distribution function of the particles in the space of invariants of the motion.

The cooling rates calculation was performed under the following general assumption: the ion coordinates are kept constant during ion interaction with cooling electron beam.

The motion of each particle is described by two Courant-Snyder invariants:

$$I_i = \frac{1+\alpha_i^2}{\beta_i} i_\beta^2 + 2\alpha_i i_\beta i'_\beta + \beta_i i'^2_\beta,\ i = x, z, \quad (3)$$

where x, z are a horizontal and vertical co-ordinates, α_i and β_i are alpha and beta functions in the cooling section. The invariant corresponded to synchrotron motion can be introduced in accordance with (2) as follows:

$$I_s = m\sigma_p^{\ 2}, \quad m = \begin{cases} 1 - \textit{coasting beam}, \\ 2 - \textit{bunched beam}. \end{cases}$$

where σ_p is the r.m.s. momentum spread. The ion coordinates and momentum inside the cooling section can be calculated in accordance with

$$x_\beta = \sqrt{I_x \beta_x} \sin\varphi, \quad x'_\beta = \sqrt{\frac{I_x}{\beta_x}}(\cos\varphi + \alpha_x \sin\varphi)$$

The same expressions are used for z co-ordinate with substitution of corresponding alpha and beta function.

$$x = x_\beta + D(\Delta p / p), \; x' = x'_\beta + D'(\Delta p / p), \; \frac{\Delta p}{p} = \sigma_p \cos\phi, \; s - s_0 = \sigma_s \sin\phi,$$

where s_0 is longitudinal coordinate of the bunch center, here $x' = p_x/p$.

A relative change of the ion momentum components after passing through the cooling section can be expressed with the following formula:

$$\begin{pmatrix} \dfrac{\delta p_x}{p} \\ \dfrac{\delta p_z}{p} \\ \dfrac{\delta p_s}{p} \end{pmatrix} = \overset{\circ}{\Phi}(x, z, s - s_0, x', z', \Delta p / p), \qquad (4)$$

where drag rates Φ can be calculated in the BETACOOL program using one of three different analytical formulae for friction force: at non magnetized [3] and at magnetized electron beam [3, 4]. User of the program can make a choice of the formula which he prefers to use in calculations. For instance, Fig.1 presents a shape of the friction force at magnetized electron beam given in [3].

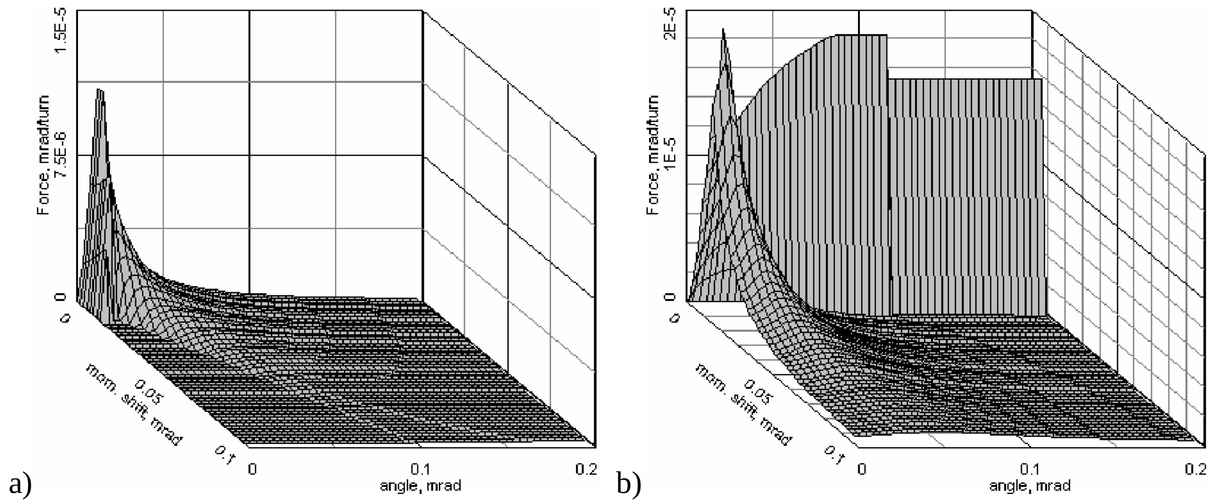

FIG.1. An example of the dependencies of the transverse (a) and longitudinal (b) components of the friction force on the ion momentum and angular shift.

The dependence of the drag rate on particle coordinates appears due to space charge effects of the ion and electron beams. Calculations are performed taking into account these effects as described in[4] [5]. To speed up the program runtime user can exclude from the calculations the ion beam space charge effects. Electron beam neutralization is taken into account under assumption of parabolic profile of the residual gas ions distribution inside the electron beam. Influence of electron beam space charge introduces and electron momentum shift and electron drift velocity – both are the functions of radial position inside the electron beam. In presence of dispersion in the cooling section an asymmetry between vertical and horizontal degrees of freedom appears. And lastlly, in the case of the intense ion bunched beam the drag rate depends on the ion distance from the bunch center.

The deviations of the invariants of motion after one passing through the cooling section are given by the following expressions:

$$\delta I_x = -2\left(x_\beta \left(\frac{\left(1+\alpha_x^2\right)}{\beta_x} D + \alpha_x D' \right) + x'_\beta \tilde{D} \right)\frac{\delta p}{p} + \\ + \frac{\left(D^2 + \tilde{D}^2\right)}{\beta_x}\left(\frac{\delta p}{p}\right)^2 + 2\left(\beta_x x'_\beta + \alpha_x x_\beta\right)\frac{\delta p_x}{p} + \quad , \\ + \beta_x \left(\frac{\delta p_x}{p}\right)^2 - 2\tilde{D}\frac{\delta p}{p}\frac{\delta p_x}{p} \tag{5}$$

$$\delta I_z = 2\left(\beta_z z'_\beta + \alpha_x z_\beta\right)\frac{\delta p_z}{p} + \beta_z \left(\frac{\delta p_z}{p}\right)^2, \tag{6}$$

$$\delta I_s = 2m\frac{\delta p}{p}\frac{\Delta p}{p} + m\left(\frac{\delta p}{p}\right)^2, \tag{7}$$

where $\frac{\delta p}{p} = \delta \frac{\Delta p}{p}$ is deviation of the relative momentum shift, D and D' are dispersion and its derivative in the cooling section, $\tilde{D} = \alpha_x D + \beta_x D'$.

The expressions (5) - (7) include the diffusion terms proportional to the square of momentum deviations. These terms determine equilibrium beam parameters in absence of other heating effects. In presence of more powerful heating processes like IBS or scattering on residual gas atoms we can ignore the diffusion effects and speed up the program runtime excluding them from the calculations.

Under assumption, that the ion distribution over betatron and synchrotron phases is uniform in an interval $[0, 2\pi]$ (a stationary beam) we can calculate average invariant deviation for ions having the same invariants of motion:

$$\left\langle \delta \mathring{I} \right\rangle = \frac{1}{8\pi^3}\int\int_0^{2\pi}\int \delta \mathring{I}\left(\mathring{I}, \varphi_x, \varphi_z, \varphi_s\right) d\varphi_x d\varphi_z d\varphi_s . \tag{8}$$

These values calculated at invariants of motion which are equal to beam emittances and can be used for evaluation of so called "single particle" cooling rate:

$$\frac{1}{\tau_{cool,sp}} = \frac{1}{I}\frac{\langle \delta I \rangle}{T_{rev}}, \tag{9}$$

where T_{rev} is the particle revolution period in the storage ring.

The cooling rates for ion beam with Gaussian distribution are calculated in BETACOOL by averaging (8) over distribution function in the space of invariants:

$$\delta\varepsilon_i = \frac{1}{\varepsilon_x\varepsilon_z\varepsilon_s}\int_0^\infty \langle \delta I_i \rangle e^{-I_x/\varepsilon_x - I_z/\varepsilon_z - I_s/\varepsilon_s} dI_x dI_z dI_s \tag{10}$$

and cooling time value is equal :

$$\frac{1}{\tau_{cool,i}} = \frac{1}{\varepsilon_i}\frac{d\varepsilon_i}{dt} = \frac{1}{\varepsilon_i}\frac{\delta\varepsilon_i}{T_{rev}}. \tag{11}$$

It should be noted, that emittance used in Formula (9) as a parameter of distribution function contains 63% of particles and is twice higher than r.m.s. value and this has to be taken into account in calculations of the standard deviation values.

Both possibilities can be used in the simulations: cooling time can be calculated in accordance with (11) or, to speed up the calculations, in accordance with (9).

B. Calculation of growth rates

Two modes of the IBS growth rate calculations are realized in the program. At smoothed lattice structure (without variation of the beta and dispersion functions) the rates are calculated by numerical evaluation of corresponding integrals in accordance with Piwinski model [6]. For real lattice of the ring the algorithm described in [7] is used.

For the smoothed focusing approximation only the mean values of the lattice functions are used and they are determined as follows:

$$\beta_{h,v} = \frac{R}{Q_{h,v}},\ D = \frac{R}{Q_h^2},\ \alpha_{h,v} = D' = 0. \tag{12}$$

In accordance with Piwinski model the growth rates are calculated in accordance with the following expressions:

$$\frac{1}{\tau_p} = \frac{1}{2\sigma_p^2}\frac{d\sigma_p^2}{dt} = nA\frac{\sigma_h^2}{\sigma_p^2} f(a,b,c)$$

$$\frac{1}{\tau_x} = \frac{1}{2\sigma_{x\beta}^2}\frac{d\sigma_{x\beta}^2}{dt} = A\left[f\left(\frac{1}{a},\frac{b}{a},\frac{c}{a}\right) + \frac{D^2\sigma_p^2}{\sigma_{x\beta}^2} f(a,b,c)\right] \tag{13}$$

$$\frac{1}{\tau_z} = \frac{1}{2\sigma_z^2}\frac{d\sigma_z^2}{dt} = Af\left(\frac{1}{b},\frac{a}{b},\frac{c}{b}\right)$$

where $n = 1$ for a bunched beam and $n = 2$ for an coasting beam,

$$A = \frac{r_i^2 c N_b}{64\pi^2\sigma_s\sigma_p\sigma_{x\beta}\sigma_z\sigma_{x'}\sigma_{z'}\beta^3\gamma^4} \text{ - bunched beam} \tag{14}$$

and for coasting beam one needs to use a substitution $\frac{N_b}{\sigma_s} \to \frac{2\sqrt{\pi}N}{C}$.

The standard deviations are determined as follows:

$$\sigma_{x_\beta,z} = \sqrt{\frac{\varepsilon_{x,z}\beta_{x,z}}{2}},\ \sigma_{x'_\beta,z'} = \sqrt{\frac{(1+\alpha_{x,z}^2)\varepsilon_{x,z}}{2\beta_{x,z}}}, \tag{15}$$

and σ_p is the r.m.s. momentum spread.

The function f is the following integral:

$$f(a,b,c) = 8\pi^2 \int_0^1 \left[\ln\left(\frac{c^2}{2} \left(\frac{1}{\sqrt{p}} + \frac{1}{\sqrt{q}} \right) \right) - 0.577 \right] \left(1 - 3x^2\right) \frac{dx}{\sqrt{pq}} . \tag{16}$$

The following relations determine normalized parameters:

$$a = \frac{\sigma_h}{\gamma \sigma_{x'}},\ b = \frac{\sigma_h}{\gamma \sigma_{z'}},\ c = \beta \sigma_h \sqrt{2 \frac{d}{r_i}},\ \frac{1}{\sigma_h^2} = \frac{1}{\sigma_p^2} + \frac{D^2}{\sigma_{x\beta}^2},$$

$$p = a^2 + x^2\left(1 - a^2\right),$$

$$q = b^2 + x^2\left(1 - b^2\right),$$

and the maximum impact parameter d is about 0.5 beam height. Integral (16) is calculated numerically.

For calculation of the IBS in accordance with [6] the lattice functions of the ring are imported from output file of the MAD program [8]. For this aim a special visual form was elaborated (Fig. 2). Using it an user can specify positions of the corresponding parameters in output FORTRAN file, read the file and check a validity of the data presented in the numerical or graphic format.

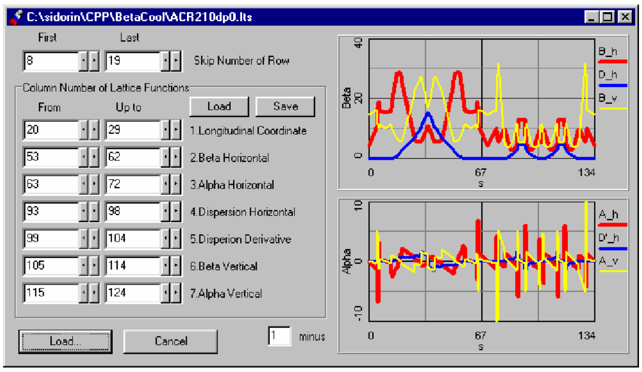

FIG. 2. The lattice parameters imported from the MAD output file.

In accordance with describing model the growth rates can be inserting in the form:

$$\frac{1}{\tau_p} = \left\langle \frac{nA}{2} \left(1 - d^2\right) f_1 \right\rangle$$

$$\frac{1}{\tau_{x'}} = \left\langle \frac{A}{2} \left[f_2 + \left(d^2 + \tilde{d}^2\right) f_1 \right] \right\rangle \tag{17}$$

$$\frac{1}{\tau_{z'}} = \left\langle \frac{A}{2} f_3 \right\rangle$$

where angular brackets mean averaging over the ring circumference, $n = 1$ for a bunched beam and $n = 2$ for coasting beam,

$$A = \frac{\sqrt{1 + \alpha_x^2}\sqrt{1 + \alpha_z^2}\, c r_i^2 \lambda}{16\pi \sqrt{\pi}\, \sigma_{x_\beta} \sigma_{x'_\beta} \sigma_z \sigma_{z'} \sigma_p \beta^3 \gamma^4} \tag{18}$$

where λ is the linear ion density:

$$\lambda = \begin{cases} N/L, & \textit{for an unbunched beam} \\ N_b/\left(2\sqrt{\pi}\sigma_s\right), & \textit{for a bunched beam} \end{cases} . \tag{19}$$

The functions f_i are integrals of the following form:

$$f_i = k_i \int_0^\infty \int_0^\pi \int_0^{2\pi} \sin\mu g_i(\mu,\nu)\exp\left[-D(\mu,\nu)z\right]\ln\left(1+z^2\right)d\nu d\mu dz , \tag{20}$$

with the coefficients $k_1 = 1/c^2$, $k_2 = a^2/c^2$, $k_3 = b^2/c^2$,
and

$$D(\mu,\nu) = \frac{\left[\sin^2\mu\cos^2\nu + \sin^2\mu\left(a\sin\nu - \tilde{d}\cos\nu\right)^2 + b^2\cos^2\mu\right]}{c^2}, \tag{21}$$

$$g_1(\mu,\nu) = 1 - 3\sin^2\mu\cos^2\nu , \tag{22}$$

$$g_2(\mu,\nu) = 1 - 3\sin^2\mu\sin^2\nu + 6\tilde{d}\sin^2\mu\sin\nu\cos\nu / a , \tag{23}$$

$$g_3(\mu,\nu) = 1 - 3\cos^2\mu . \tag{24}$$

The normalized parameters are calculated from the following expressions:

$$a = \frac{\sigma_y}{\sigma_{x'_\beta}}\sqrt{1+\alpha_x^2} ,\ b = \frac{\sigma_y}{\sigma_{z'}} ,\ c = q\sigma_y ,\ d = \frac{\sigma_p}{\sigma_x}D ,\ \tilde{d} = \frac{\sigma_p}{\sigma_x}\tilde{D} ,$$

where $\tilde{D} = \alpha_x D + \beta_x D', \sigma_x^2 = \sigma_{x_\beta}^2 + D^2\sigma_p^2$, $\sigma_y = \frac{\sigma_p \sigma_{x_\beta}}{\gamma\sigma_x}$ and $q = 2\beta\gamma\sqrt{\frac{\sigma_z}{r_i}}$.

The integration over z – variable can be approximately performed analytically due to small value of the function $D(\mu,\nu)$:

$$\int_0^\infty \exp\left[-D(\mu,\nu)z\right]\ln\left(1+z^2\right)dz \approx 2\int_1^\infty \exp\left[-D(\mu,\nu)z\right]\ln z dz =$$

$$-\frac{2}{D(\mu,\nu)}\left(C + \ln D(\mu,\nu) + \int_0^{D(\mu,\nu)} \frac{e^{-t}-1}{t}dt\right).$$

The integration (20) over other two variables is performed numerically.

C. Calculations of the beam stability characteristics

Incoherent tune shift is calculated using well known formula:

$$\Delta Q_{sc} \approx \frac{r_i}{2\pi}\frac{N_b}{\beta^2\gamma^3\varepsilon_z(1+\sqrt{\varepsilon_x/\varepsilon_z})B_f} , \tag{25}$$

where N_b is the particle number in the case of coasting beam and product of the particle number per bunch and harmonic number in the case of bunching beam, B_f - bunching factor (relation of average current to peak current).

As a characteristics of a longitudinal stability of the beam the following parameter are calculated:

$$KS = \left(4F_L \frac{U_p}{(|Z_L|/n)} \frac{A_i}{Z_i I} \gamma\beta^2 |\Gamma| (\frac{\Delta p}{p})^2 \right)^{-1} \le 1. \qquad (26)$$

Here F_L is a factor depending on form of distribution function and the impedance phase (in "Keil-Schnell" criterion it is assumed that F_L =1), U_p = 938 MV, $\Gamma = 1/\gamma^2 - 1/\gamma_{tr}{}^2$, Z_n is the longitudinal coupling impedance at $\omega = n\omega_0$.

Criterion for transverse beam stability is calculated as follows

$$SZ = \left(8F_t \frac{U_p}{Z_t R} \frac{A_i}{Z_i I} \beta\gamma Q \Delta Q_n \right)^{-1} \le 1. \qquad (27)$$

Here effective spread of betatron tune for a mode with n-th number ΔQ_n is defined by

$$\Delta Q_n \approx \sqrt{[(n-Q)\Gamma + \xi]^2 \left(\frac{\Delta p}{p} \right)^2 + \Delta Q_{non}^2}\ , \qquad (28)$$

Z_t is transverse coupling impedance at $\omega = (n-Q)\omega_0$, and ξ is the rind chromaticity, F_t is a factor depending on form of distribution function and the impedance phase.

At the regime of the storage ring with the bunched ion beam the influence of the space charge fields on synchrotron motion is estimated during calculations also. For calculation of linear synchrotron tune Q_s the model of the beam with linear longitudinal space charge field and parabolic dependence of line charge density on longitudinal coordinate is applied. The synchrotron tune is calculated as follows

$$Q_s = Q_s{}^0 \sqrt{1 - N_b / N_b{}^{\max}}\ , \qquad (29)$$

where N_b is ion number per bunch, its maximal value - N_b is given by expression:

$$N_b{}^{\max} = \frac{1}{3\pi G_L} \cdot \frac{q(eU)\sigma_s{}^3\gamma^2}{Z_i E_0 r_p R^2}\ , \qquad (30)$$

$G_L = 1 + 2\ln(\frac{b}{a})$, U is RF voltage, q is RF harmonic number, E_0 and r_p are the proton energy and classical radius, Z_i is an ion charge number. The tune $Q_s{}^0$ appearing due to RF field is

$$Q_s{}^0 = \frac{1}{\beta} \sqrt{\frac{q\Gamma e Z_i U}{2\pi E_0 A_i \gamma}}\ , \qquad (31)$$

here A_i is the ion atomic number, β and γ are usual relativistic factors. R.m.s momentum spread is connected to σ_s by

$$\sigma_p = Q_s \frac{\sigma_s}{\Gamma R}. \qquad (32)$$

Now for calculation of the bunch length the tune Q_s^0 is used, the value (28) is displayed as well as the values (25), (26) and (27) in graphic form for estimation of the influence of the beam space charge on the particles motion.

D. Luminosity calculations

Luminosity calculations are performed for the bunches which cross each other with the angle of 2φ in horizontal (or vertical) plane (Fig. 3) and assume that they have the Gaussian particle density distribution in own frame:

$$\rho_1(x_1, z_1, s_1, v_1, t) = \frac{N_1}{(2\pi)^{3/2}\sigma_{x1}\sigma_{zy1}\sigma_{s1}} \exp\left(-\frac{x_1^2}{2\sigma_{x1}^2} - \frac{z_1^2}{2\sigma_{z1}^2} - \frac{(s_1 - v_1 t)^2}{2\sigma_{s1}^2}\right),$$
$$\rho_2(x_2, z_2, s_2, v_2, t) = \frac{N_2}{(2\pi)^{3/2}\sigma_{x2}\sigma_{z2}\sigma_{s2}} \exp\left(-\frac{x_2^2}{2\sigma_{x2}^2} - \frac{z_2^2}{2\sigma_{z2}^2} - \frac{(s_2 - v_2 t)^2}{2\sigma_{s2}^2}\right). \quad (33)$$

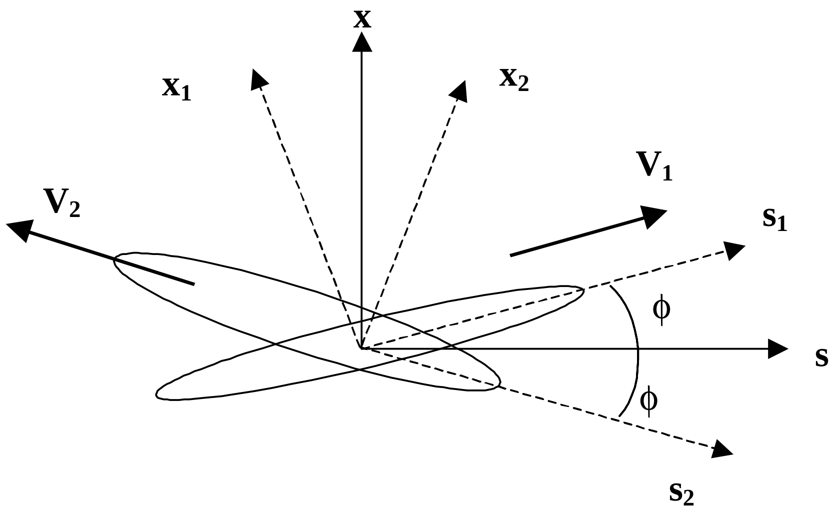

FIG.3. Schematics of the beam-beam interaction at a crossing angle

Luminosity can be expressed by the following formula:

$$L = N_b f_0 c \frac{N_1 N_2}{(2\pi)^3} F(\beta_1, \beta_2, \varphi) \cdot I(\sigma_{x1}, \sigma_{z1}, \sigma_{s1}, \sigma_{x2}, \sigma_{z2}, \sigma_{s2}, v_1, v_2), \quad (34)$$

where $F(\beta_1, \beta_2, \varphi) = \sqrt{\beta_1^2 + \beta_2^2 + 2\beta_1\beta_2\cos(2\varphi) - \beta_1^2\beta_2^2\sin(2\varphi)}$ is kinematics factor, f_0 is particle revolution frequency in the ring, and integral over volume and time of interaction calculated analytically over time, x and z coordinates is the following:

$$I = \int_{-\infty}^{\infty} \frac{(2\pi)^{3/2}}{c\sqrt{\beta_1^2\sigma_{s2}^2 + \beta_2^2\sigma_{s1}^2}\sqrt{\sigma_{z1}^2 + \sigma_{z2}^2}\sqrt{\sigma_{x1}^2 + \sigma_{x2}^2}\sqrt{\cos^2\varphi + \sin^2\varphi\frac{\sigma_{x1}^2\sigma_{x2}^2(\beta_1 - \beta_2)^2}{(\sigma_{s1}^2 + \sigma_{s2}^2)(\sigma_{s2}^2\beta_1^2 + \sigma_{s1}^2\beta_2^2)}}} \times$$
$$\times \exp\left\{-\frac{s^2}{2}\left[e - \frac{b_2^2}{a} - \frac{(b_1 b_2 + ad)^2}{a(ac - b_1^2)}\right]\right\} ds, \quad (35)$$

where

$$a=\frac{\beta_1^2}{\sigma_{s1}^2}+\frac{\beta_2^2}{\sigma_{s2}^2},$$

$$b_1=\sin\varphi\left(\frac{\beta_1}{\sigma_{s1}^2}+\frac{\beta_2}{\sigma_{s2}^2}\right),$$

$$b_2=\cos\varphi\left(-\frac{\beta_1}{\sigma_{s1}^2}+\frac{\beta_2}{\sigma_{s2}^2}\right),$$

$$c=\cos^2\varphi\left(\frac{1}{\sigma_{x1}^2}+\frac{1}{\sigma_{x2}^2}\right)+\sin^2\varphi\left(\frac{1}{\sigma_{s1}^2}+\frac{1}{\sigma_{s2}^2}\right),$$

$$d=\sin\varphi\cos\varphi\left(-\frac{1}{\sigma_{x1}^2}+\frac{1}{\sigma_{x2}^2}+\frac{1}{\sigma_{s1}^2}-\frac{1}{\sigma_{s2}^2}\right),$$

$$e=\sin^2\varphi\left(\frac{1}{\sigma_{x1}^2}+\frac{1}{\sigma_{x2}^2}\right)+\cos^2\varphi\left(\frac{1}{\sigma_{s1}^2}+\frac{1}{\sigma_{s2}^2}\right). \quad (36)$$

This integral is calculated numerically taking into account that in the vicinity of interaction point the bunch sizes are the functions of longitudinal coordinate:

$$\sigma_{xi}^2=\sigma_{xi}^{*2}\left(1+\frac{s^2}{\beta_{xi}^{*2}}\right),\ \sigma_{zi}^2=\sigma_{zi}^{*2}\left(1+\frac{s^2}{\beta_{zi}^{*2}}\right), \quad (37)$$

where σ^* and β^* are corresponding beam size and beta function at the interaction point.

To optimize the lattice parameters at the interaction point a special form was elaborated. It presents the luminosity dependence on the beta functions in the interaction point as a three dimensional plot under assumption that $\beta_{x1}^*=\beta_{z1}^*$ and $\beta_{x2}^*=\beta_{z2}^*$. As example in the Fig. 4 the results of calculation of luminosity in the collisions of electron and uranium bunches in the DSR ring of the MUSES project (RIKEN, Japan) is presented.

If the collision mode of the ring operation is chosen the luminosity is displayed in graphic format during calculations.

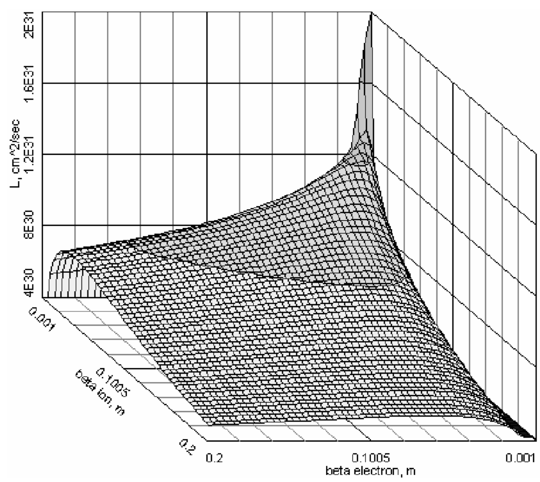

FIG.4. Example of the calculation of the luminosity dependence on beta functions in the interaction point (e-ion collider).

3. DESCRIPTION OF THE PROGRAM

The BETACOOL program is developed on the base of BOLIDE system using C++ Builder3 and works now under Microsoft Windows operation system. The BOLIDE version for Unix is under design. In the program only the standard C++ and BOLIDE commands are used for connection of algorithmic and interface parts. In the future the program can be recompiled to Unix version without any modifications.

The numerical algorithm is realized on the base of Object Oriented Programming method and the program structure consists of several basic objects. For input and visual presentation of parameters of each object special forms were developed. The tools of the BOLIDE system to load and save data into hard disc, for output the data into two and tree dimensional plots during the calculations and to control of the calculation process are used. Below we discuss briefly general objects of the program.

The object `Ring` includes the general parameters of the storage ring, methods for import of the parameters from external file and control of its validity, methods for calculation of required parameter. The parameters are divided by several groups in accordance with general systems of the storage ring: parameters of the stored ions, lattice parameters mean value and imported from external file, radio frequency and vacuum system parameters. View of the form for input of the ring parameters is presented in the Fig.5a.

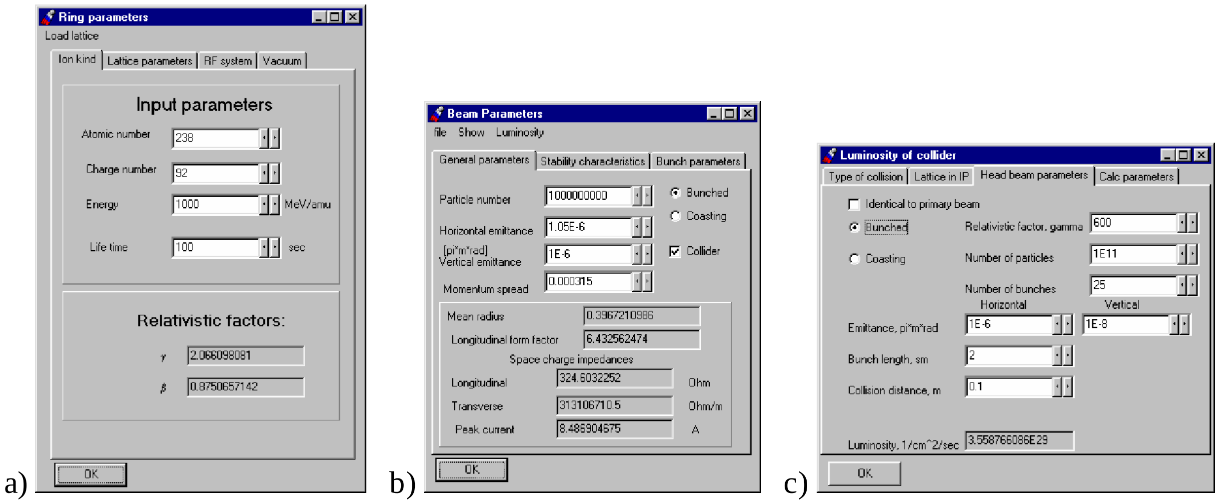

FIG.5. a) The form dedicated to input and output the parameters of the storage ring;
b) The form dedicated to input and output the beam parameters in the numerical format.
c) The form dedicated to input and output the parameters required for luminosity calculation.

The object `Beam` includes general beam parameters: particle number, emittances and momentum spread values, methods for calculation of beam parameters characterizing beam stability and luminosity, for of general parameters visualization in numerical and graphic form. The methods use only ring parameters as input. Fig.5, 6a present the forms for input and output of the beam parameters.

Each process involved into calculation of beam parameters evolution is performed as an independent object also. This object includes the parameters of corresponding process or device and methods dedicated to optimization of these parameters and their visualization. For instance, object "Electron Cooling" includes parameters of the cooling section, parameters of the electron beam and lattice functions in the cooling section. To optimize cooling time calculations user can choose physical model of the process and mode of the cooler operation. The dependence of friction force on ion angle and momentum deviation is outputted as a three dimensional plot, and for mode with circulating electron beam the time dependence of electron temperature is displayed in graphic format also. The view of the form dedicated to input of the electron cooling system parameters is presented in the Fig.6b.

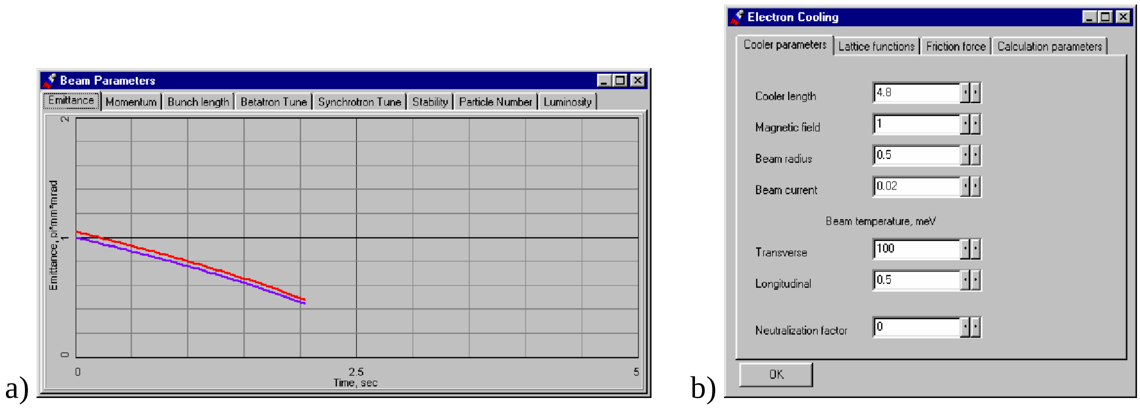

FIG. 6. a) The form dedicated to output the beam parameters in the graphic format.
b) The form dedicated to input the electron cooling system parameters.

All objects describing a beam parameters evolution are developed on the base of the ancestor class `Effect`, which has a virtual function using `Beam` and `Ring` objects as input variables, and returning array of the rates: particle loss, two emittances and momentum spread variation. In each descendants class corresponding to concrete process this function is reloaded by its individual one. All variables of the class effect are automatically included into array of the effects, and class effect includes the method, which calculates the sum of the rates in the cycle. This ancestor class includes also methods for output of the rates in numerical and graphical format (Fig.7).

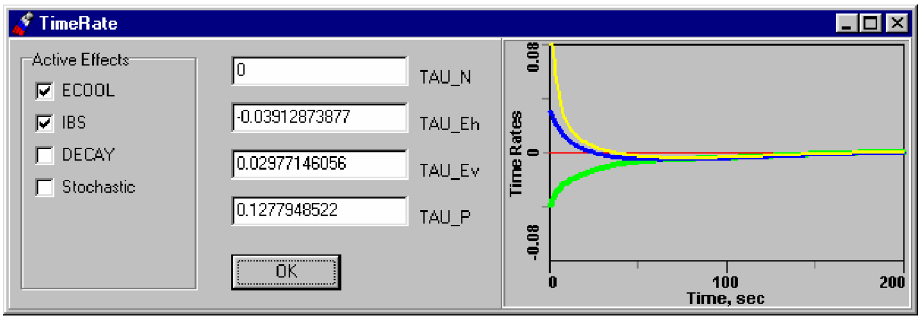

FIG.7. The form dedicated to visualization of the effect array and output of the sum of the rates of the active effects.

In order to realize this approach the special class `BTemplate` for the storage of object pointers was elaborated in the program. The declaration and constructor of this template are presented here:

```
template <class B>
class BTemplate
{ public:
   static B **BItems;
   static int BCount;
   int BIndex;
   BTemplate();
   virtual ~BTemplate();
};

template <class B>
BTemplate<B>::BTemplate()
{ B** newItems = new B*[BCount+1];
   for (int i = 0; i < BCount; i++)
      newItems[i] = BItems[i];
   newItems[BCount] = (B*)this;
   if (BCount) delete []BItems;
   BItems = newItems;
   BIndex = BCount;
   BCount++;
}
```

All `Effect` classes are the descendants from `BTemplate`. Simulation tasks do not use standard variable names of different `Effect` classes. They use the pointer array `BItems` of `BTemplate`. This structure permits to include new effects without any changes in the program very easy. Such an approach is useful and widely used in our program codes as the base of Object Oriented Programming.

Another group of objects called “Task” corresponds to investigation of the beam parameter variation due to common action of the active effects (Fig.7). These objects get the array of the rates from the list of effects, calculate the beam parameters and use the methods of the `Beam` object for their visualization. One object of this type only is realized now in program – “Dynamics”, which includes method and parameters necessary for numerical solution of the system of differential equations (1) and for visualization of the results of calculations. The “Task” dedicated to calculation of the equilibrium state of the ion beam is under development presently.

CONCLUSION

In the presented version of the program a calculation of following processes was realised:

- intra beam scattering (IBS),
- electron cooling (ECOOL),
- decay of unstable nuclei.

BETACOOL structure is designed as Object Oriented Program that permits to include into calculation each process, which can be described by cooling or heating rates and characteristic times of the particle number variation. In the frames of the described physical model we can provide a common simulation also of the following processes:

- stochastic cooling,
- ion scattering on residual gas atoms,
- ion beam interaction with internal target,
- ion beam heating due to non-linear resonances,
- particle losses related to different factors,
- particle injection and so on.

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