Some Recent Results In Numerical Simulation Of Crystallisation With BETACOOL Code

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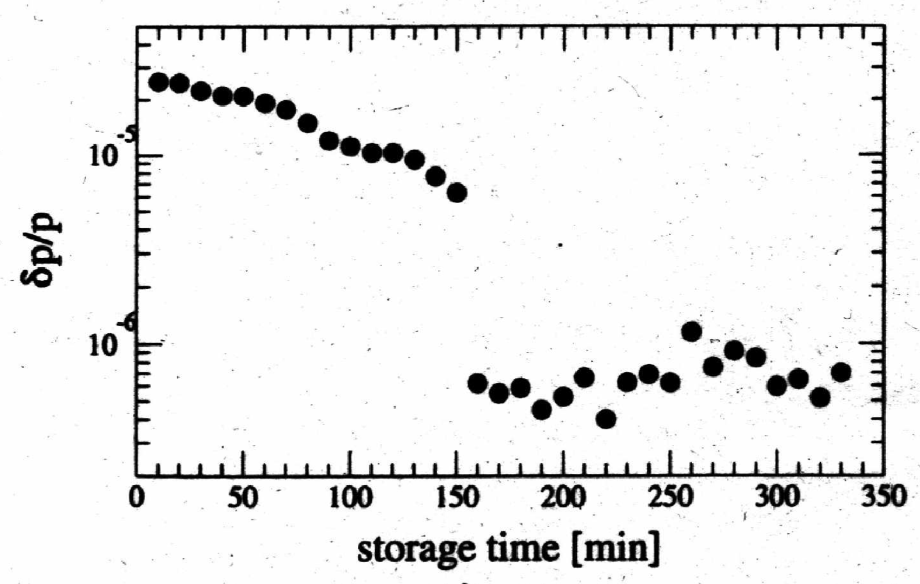


FIG. 1. Schottky noise power and momentum spread (FWHM) during decrease of beam intensity due to radiative recombination in the electron cooler.

ESR experiment

PLASMA PARAMETER

$$\Gamma = \frac{U}{T} = \frac{Z^2 e^2}{aT} > 1$$

where U and T - potential energy and temperature of ion beam, Ze - charge of particles, a - average particle distance.

$$\Gamma = \frac{Z^2 e^2}{T_\perp} \left(\frac{N_i}{\varepsilon_\perp < \beta^* > \sigma_\parallel} \right)^{1/3} > 1 \text{ , when } T_\perp >> T_\parallel$$

where ρ - particle density, $<\sigma_\perp>$ - average transverse ion size, $\sigma_\parallel$ - bunch length, $\varepsilon_\perp$ - transverse emittance, $<\beta^*>$ - average beta function along a ring, N_i - number of ions per bunch.

$$T_\perp = M_i c^2 \beta^2 \gamma^2 \theta^2_{rms}$$

where $M_i c^2$ - ion rest mass, β and γ -relativistic factors, $\theta^2_{rms} = \varepsilon / <\beta>$ - transverse beam divergence.

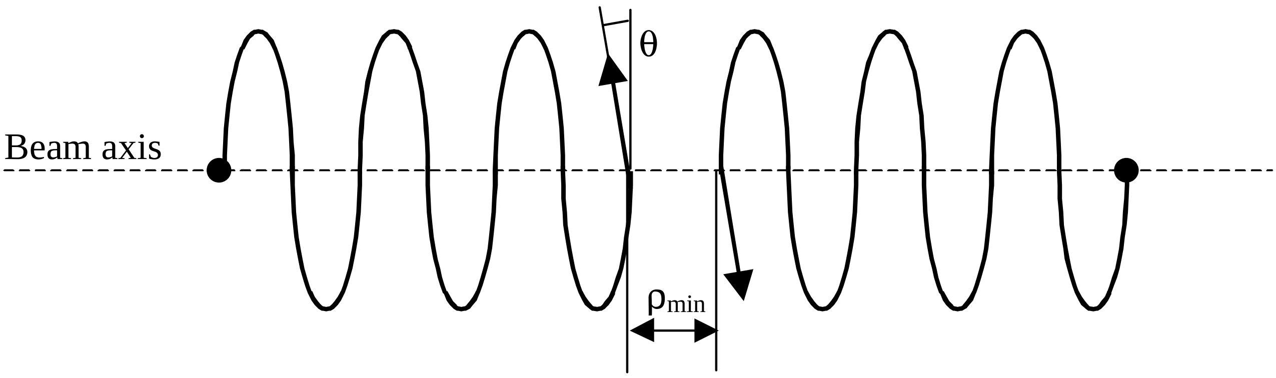
$$T_\parallel = M_i c^2 \beta^2 (\delta p/p)^2_{rms}$$

where $(\delta p/p)_{rms}$ - rms momentum spread, which can be determined from measured momentum spread with $(\delta p/p)_{rms} = (8 \times \ln 2)^{-1/2} (\delta p/p)$.

As a rule, ion transverse temperature is much than the longitudinal one:

$$T_\perp >> T_\parallel$$

ORDERED BEAM CONDITIONS



1) The scattering angle has to be so large that the longitudinal component of the scattered ion change its sign and the ion moves backwards keeping (in average) its position in the chain. It means that

$$\theta\, V_{\perp} > V_{||}\ .$$

Necessary condition of the beam ordering:

$$\Gamma_1 = \frac{Z^2 e^2}{a_{\|}\sqrt{T_{\|} T_{\perp}}} < 1\ .$$

2) Ions do not pass each other without an appropriate scattering by an angle θ:

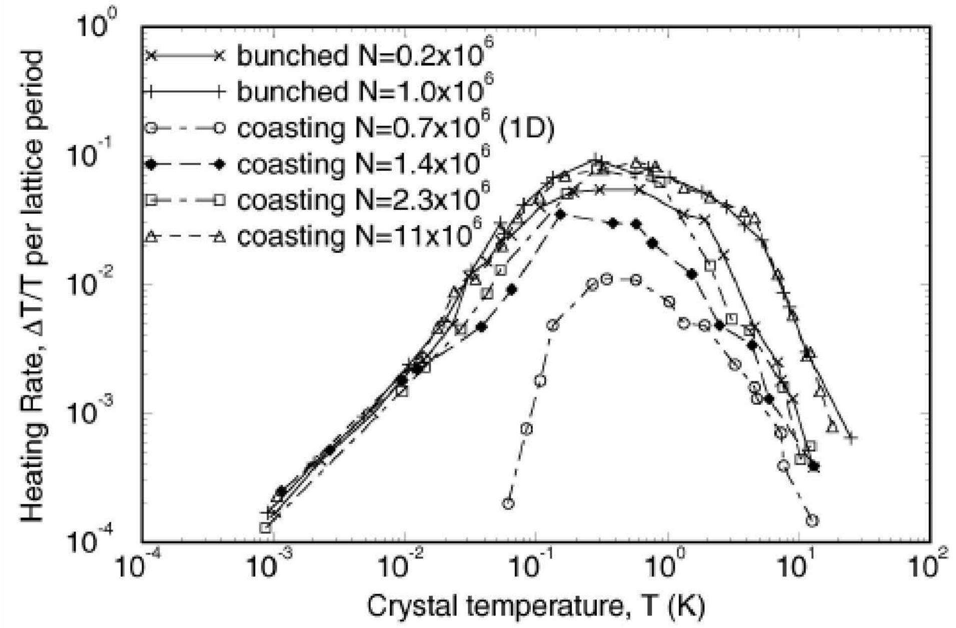
$$(\Delta t)_{\rho} \geq \frac{\tau_{bet}^{PRF}}{4}\ .$$

where $(\Delta t)_{\rho} \sim \dfrac{\rho_{\min}}{V_{\|}}$, $\quad \tau_{bet}^{PRF} = \dfrac{C_{Ring}}{Q_{bet}\gamma\beta c}$

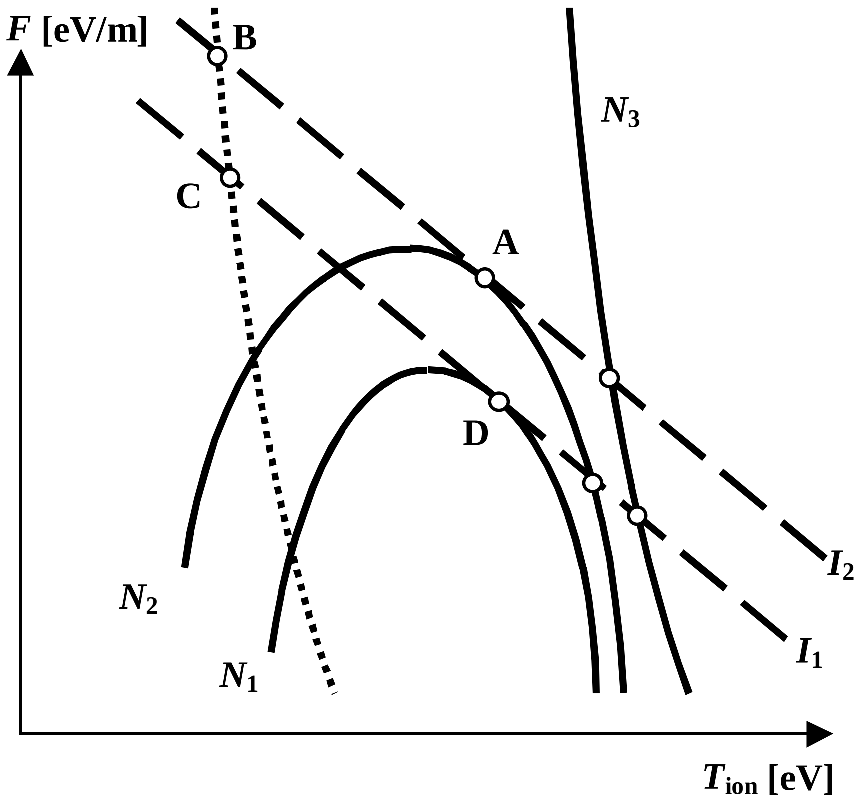
And second sufficient condition is:

$$\Gamma_2 = \frac{Z^2 e^2}{T_{\|}\sigma_{\perp}} > \pi\ .$$

Heating rate curves for bunched beams and coasting beams at different linear density (SOILD program simulation, BNL, USA)



Dependence of cooling and heating forces on the ion temperature. Solid line corresponds to IBS, dashed line - ECOOL, dotted line - other heating process.



CRYSTALLIZATION CONDITIONS

1) First condition is related to an optical structure of the storage ring. The storage ring must be alternating-gradient (AG) focusing, and energy of beam must be less than the transition energy of the ring:

$$\gamma < \gamma_T$$

This condition arises from the criterion of stable kinematics motion under Coulomb interaction, when particles are subject to bending in a storage ring.

2) The ring lattice periodicity is at least $2^{3/2} = 2.828$ as high as the maximum betatron value:

$$2\sqrt{2} \cdot \max\{v_x, v_y\} < \text{lattice periodicity}$$

This condition arises from the requirement that there is no linear resonance between the phonon modes of the crystalline structure and the machine lattice periodicity. Note that this is a condition for the maintenance of a crystalline beam, in contrast to the previous condition, which is for the formation of a crystalline beam.

EQUATIONS OF MOTION

Vector of canonical variables

$$\overset{\circ}{V} = \left(x,\ p_x = \frac{P_X}{P_S},\ y,\ p_y = \frac{P_Y}{P_S},\ z = -(t - t_0)\beta_0 c,\ \delta_z = \frac{E - E_0}{P_S \beta_0 c} \right)$$

Hamiltonian

$$H = \frac{1}{2}\left(p_x^2 + p_y^2 + \frac{\delta_z^2}{\gamma_0^2} \right) - \frac{x}{R}\delta_z + \frac{x^2}{2R^2} + \frac{K_q}{2}\left(x^2 - y^2\right) +$$
$$\frac{K_s}{6}\left(x^3 - 3xy\right) + \frac{r_{ion}}{\gamma_0^2\beta_0^2}\sum_i \left[(x - x_i)^2 + (y - y_i)^2 + \gamma_0^2 (z - z_i)^2\right]^{-1/2}$$

Cooling forces $\overset{\circ}{F}_{ec} = \left(0,\ -\frac{K_x}{L_{ec}} p_x,\ 0,\ -\frac{K_y}{L_{ec}} p_y,\ 0,\ -\frac{K_z}{L_{ec}} \delta_z \right)$

where

R	- bending radius
$K_q = \frac{1}{BR}\frac{\partial B_y}{\partial x}$	- quadruple gradient
$K_s = \frac{1}{BR}\frac{\partial^2 B_y}{\partial x^2}$	- sextuple gradient
$K_{x,y,z} = 2T_{rev} \times \Omega_{x,y,z}$	- e-cooling strengths
T_{rev}	- revolution period of particle
$\Omega_{x,y,z}$	- e-cooling rates
L_{ec}	- e-cooling system length
U_{sc}	- space charge potential

CONSTANT FOCUSING SIMULATION

Hamiltonian for constant focusing optics

$$H = \frac{1}{2}\left(p_x^2 + p_y^2 + \frac{p_\tau^2}{\gamma_0^2} \right) + \frac{K}{2}\left(x^2 + y^2\right) + U_{sc}$$

Vector of forces (including of e-cooling forces)

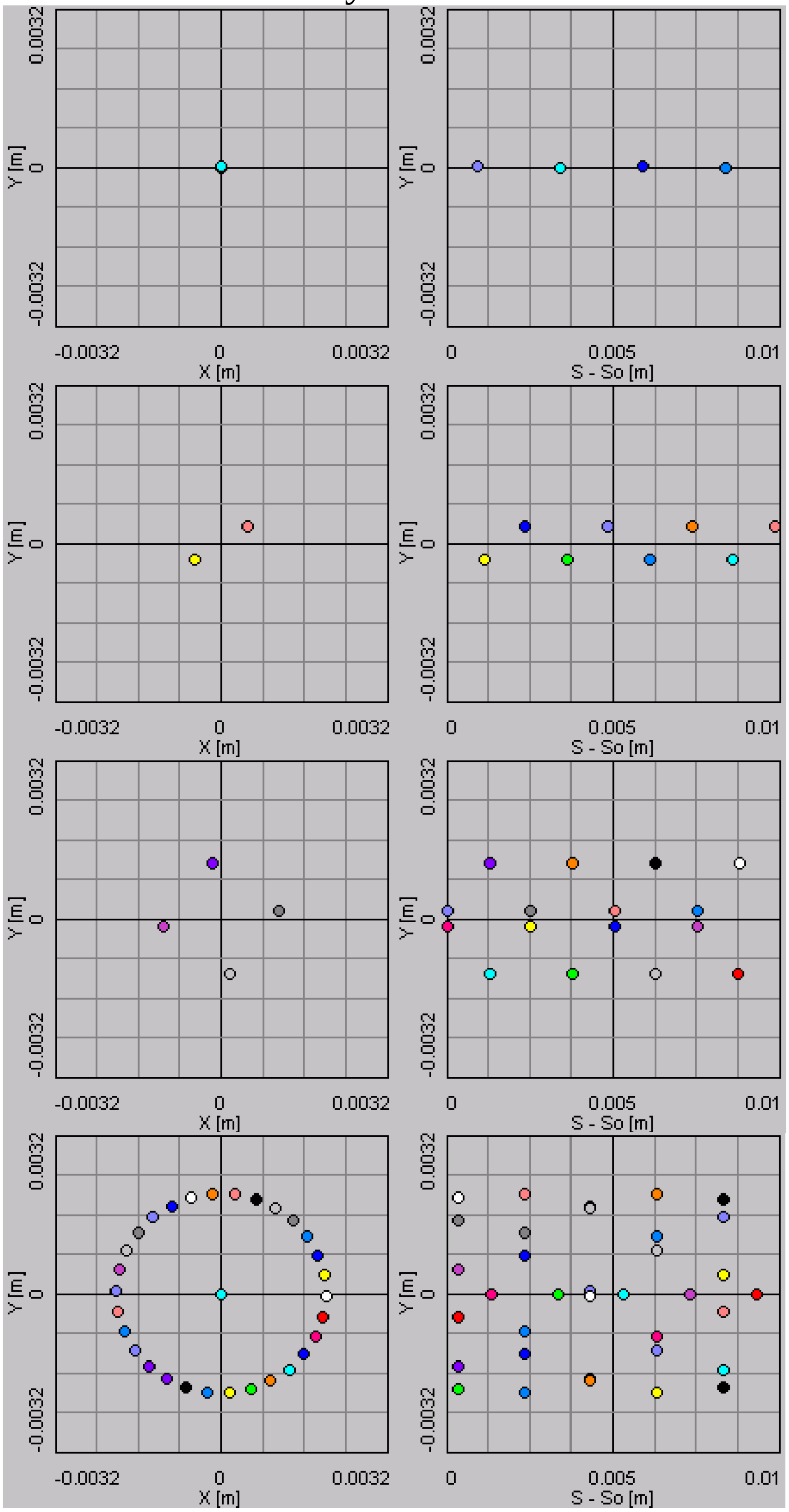
$$\overset{\circ}{F} = \begin{pmatrix} \frac{dx}{ds} \\ \frac{dp_x}{ds} \\ \frac{dy}{ds} \\ \frac{dp_y}{ds} \\ \frac{dz}{ds} \\ \frac{d\delta_z}{ds} \end{pmatrix} = \begin{pmatrix} p_x \\ -Kx - \frac{\partial U_{sc}}{\partial x} - \frac{K_x p_x}{L_{ec}} \\ p_y \\ -Ky - \frac{\partial U_{sc}}{\partial y} - \frac{K_y p_y}{L_{ec}} \\ \frac{\delta p_z}{\gamma_0^2} \\ -\frac{\partial U_{sc}}{\partial z} - \frac{K_z \delta_z}{L_{ec}} \end{pmatrix}$$

Circumference = 10 cm

E = 60 eV/u
$^{16}O^{8+}$
Periodicity = ∞
Particle per cell = 4, 8, 16, 35

ECOOL in whole channel
Step = 1 mm

Kx = Ky = Kz = $\mathbf{10^{-3}}$



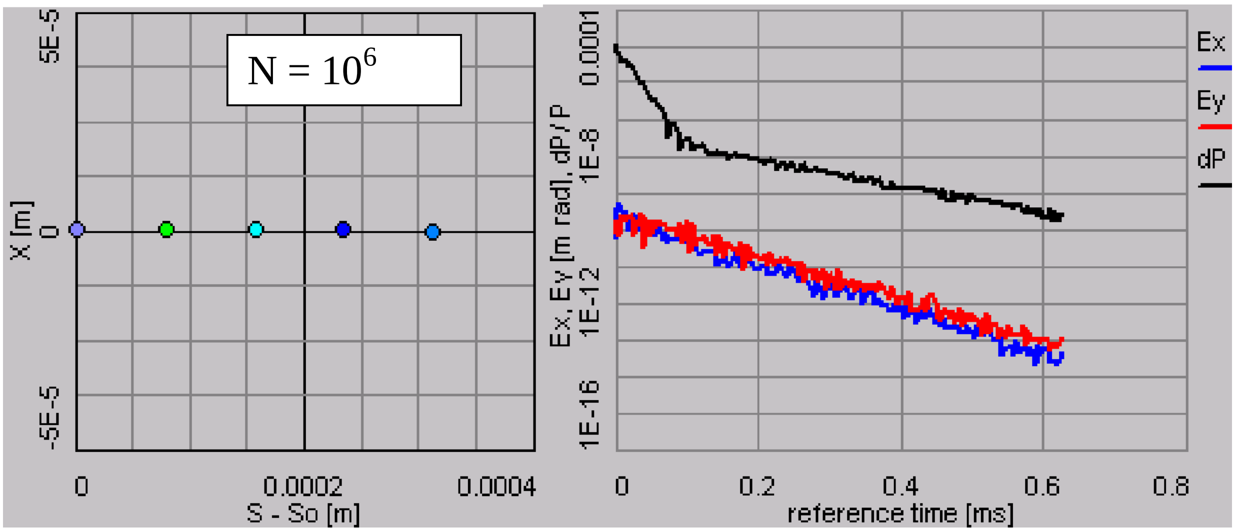
ORDERED STATE

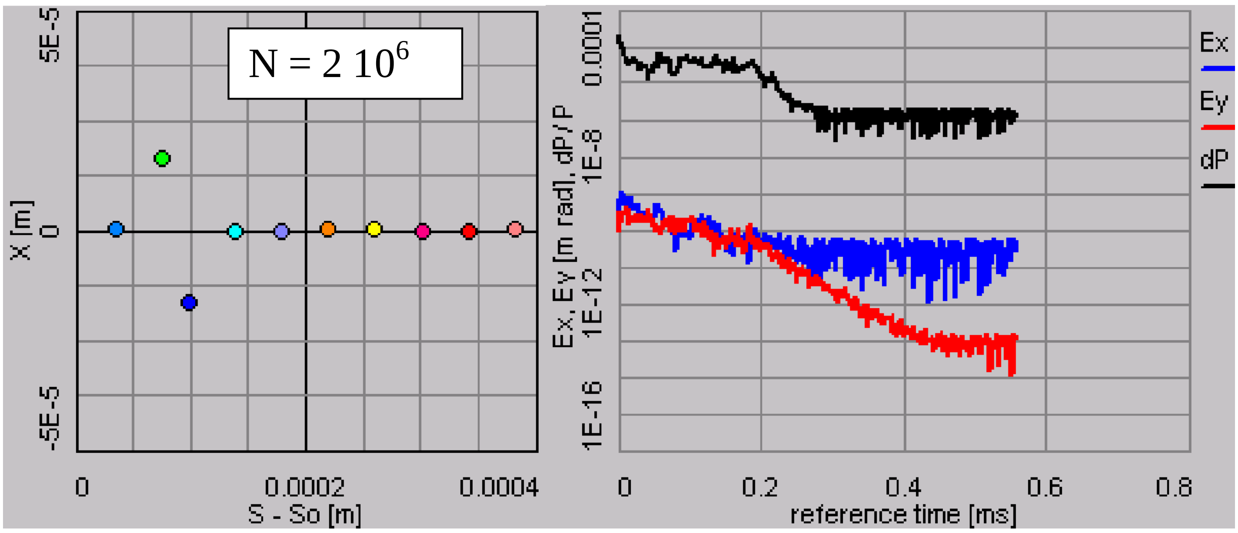
TARN-II

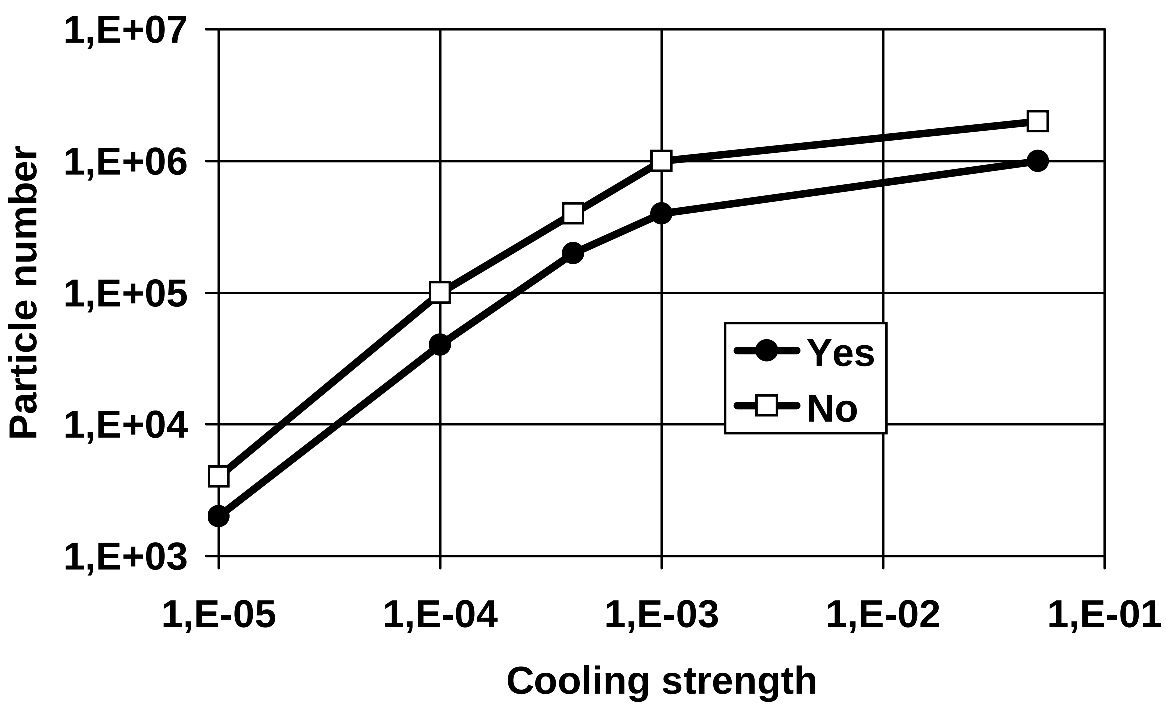
Circumference = 78 m
Qx = 1,665
Qz = 1,809
γ_{tr} = 1,88
only one ECOOL in ring (length 1.5 m)
Step = 1cm

E = 360 MeV/u
$^{179}Au^{79+}$
Periodicity = 6
Particle per cell = 10

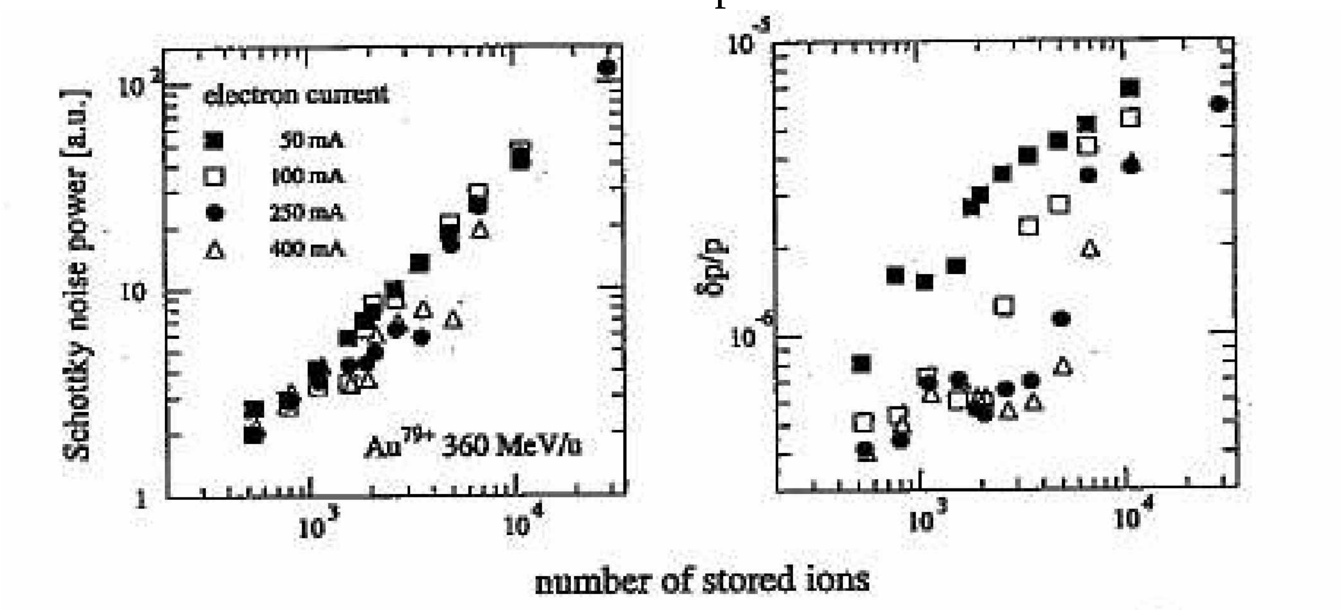
Kz = **5 10^{-2},** Kx = Ky = **5 10^{-3}**



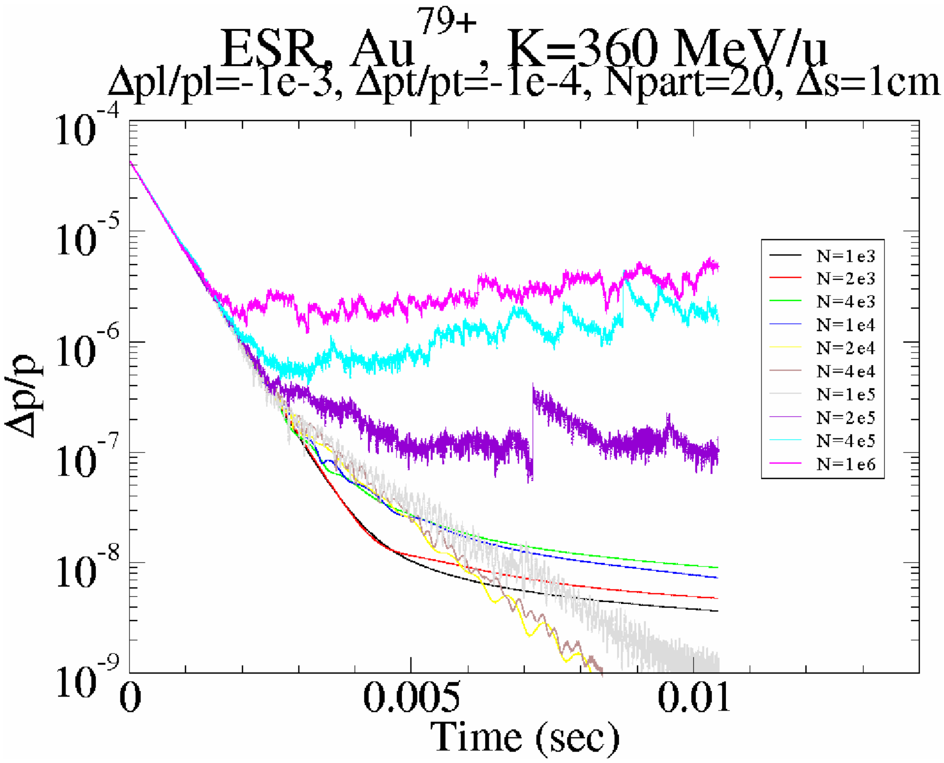




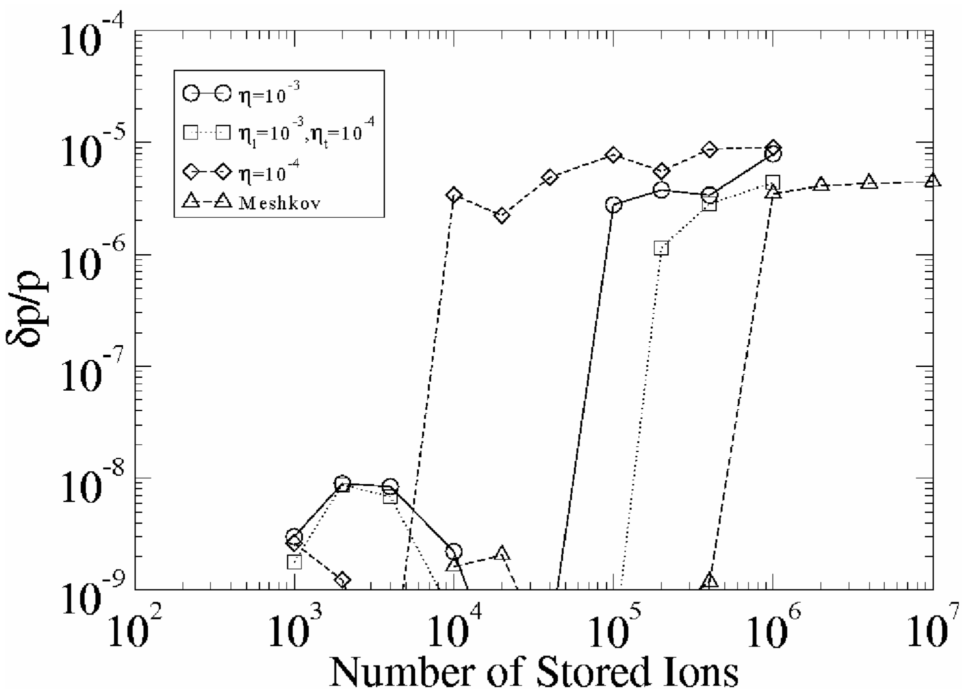
The dependence of particle number on the longitudinal cooling strength (transverse ones in 10 times less), when ordered state exist or not for TARN-II optics structure



Momentum spread reduction for different ECOOL current
ESR experiment



Simulation result of ESR experiment.
At EC section, $Kz=10^{-3}$, $Kx=Ky=10^{-4}$ are assumed.



Simulated momentum spread as a function of number of stored ions.

TAPERED COOLING

FODO Structure

Circumference = 140 m
Qx = 2,55
Qz = 2,59
γ_{tr} = 2,53

E = 360 MeV/u
$^{179}Au^{79+}$
Periodicity = 20
Particle per cell = 32, 64

Lets introduce the dependence of cooling force on horizontal coordinate

$$\Delta\delta_z = -K_z\left(\delta_z - \gamma_0 C_{xs}\frac{x}{R}\right)$$

where *R* - bending radius, C_{xs} – tapered factor

Longitudinal force for tapered cooling

$$\Delta\delta_z = -F_z s = -K_z\frac{s}{L_{cool}}\left(\delta_z - \gamma_0 C_{xs}\frac{x}{R}\right)$$

$$F_z = \frac{K_z}{L_{cool}}\left(\delta_z - \gamma_0 C_{xs}\frac{x}{R}\right)$$

Optimum tapered factor

$$\overline{C}_{xs} \approx 2\pi\frac{\gamma_0 R}{C_{ring}}$$

$$F_z = \frac{K_z}{L_{cool}}\left(\delta_z - 2\pi\gamma_0^2\frac{x}{C_{ring}}\right) = \frac{K_z}{L_{cool}}\left(\delta_z - C_t\gamma_0^2 x\right)$$

For FODO test structure

$$C_t = \frac{2\pi}{C_{ring}} = \frac{1}{R_{ring}} = \frac{2\pi}{140\,[m]} = 0.045\,\left[m^{-1}\right]$$

3D CRYSTALLINE BEAMS WITH TAPERED COOLING

Kz = **0,02**, Kx = Ky = **0,1**