NUMERICAL SIMULATION OF PARTICLE DYNAMICS IN STORAGE RINGS USING BETACOOL CODE

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Abstract

Numerical simulation of particle dynamics in storage rings is one of key issues in accelerator physics. Now a lot of programs are developed for the simulation of charged particle dynamics. The main goal of BETACOOL code [1] is simulation of beam parameters in presence of electron cooling together with other physical effects which effect on circulating beam: intrabeam scattering, internal target, beam-beam effect, scattering on residual gas, etc. This code is being developed since 1996 [2] and is used in different science centres which have electron cooling system under operation or are developing the new project of these systems.

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INTRODUCTION

An electron cooling method is widely used for ion beam parameter control in storage rings. Presently there are more than 20 storage rings in operation and under construction, which are equipped with electron cooling devices. The BETACOOL program developed for simulation of electron cooling process is actively used in several research centers: FZJ, GSI (Germany), RIKEN, NIRS (Japan), BNL (USA), JINR, ITEP (Russia) [3-8]. The BETACOOL is programmed with object oriented method using C++ language. Interface part for Windows operation system is developed on the basis of BOLIDE system (Builder Object Library & Interface Development Environment) [1], which is dedicated to fast elaboration of the physics and mathematics applications.

General goal of the BETACOOL program is to simulate long term processes (in comparison with the ion revolution period) leading to variation of the ion distribution function in 6 dimensional phase space. The ion beam motion inside the ring is supposed to be stable and is treated in linear approximation.

Structure of the program permits to simulate ion distribution function evolution using a few independent numerical algorithms. Each algorithm simulates the ion beam dynamics at the same input beam and ring parameters and uses in simulations the same set of effects acting on the beam distribution function.

This report discusses last version of the BETACOOL program, which includes three algorithms for beam dynamics simulation and takes into account the following processes: electron cooling, intrabeam scattering, ion scattering on residual gas atoms, interaction of the ion beam with internal target and some others.

BASE ALGORITHMS

Three basic algorithms for simulations of the ion distribution function evolution are realized in the program now:

- RMS (root mean square) dynamics simulation,
- Simulation of distribution function evolution using Monte-Carlo method (Model Beam algorithm),
- Multi particle tracking based on Molecular Dynamics technique.

The physical model used in RMS dynamics simulation is based on the following general assumptions:

1) the ion beam has Gaussian distribution over all degrees of freedom, and is not changed during the process.

2) algorithm for analysis of the problem is considered as a solution of the equations for RMS values of the beam phase space volumes of three degrees of freedom.

3) maxima of all the distribution functions coincide with equilibrium orbit.

The evolution of the ion beam parameters during its motion inside the storage ring is described by the following system of four differential equations:

$$\begin{cases} \dot{N} = N\sum_j \dfrac{1}{\tau_{life,j}}, \\ \dot{\varepsilon}_h = \varepsilon_h \sum_j \dfrac{1}{\tau_{h,j}}, \\ \dot{\varepsilon}_v = \varepsilon_v \sum_j \dfrac{1}{\tau_{v,j}}, \\ \dot{\varepsilon}_{lon} = \varepsilon_{lon} \sum_j \dfrac{1}{\tau_{lon,j}}, \end{cases} \qquad (1)$$

where N is the particle number, ε_h, ε_v, ε_{lon} are root mean square values of horizontal, vertical and longitudinal beam emittance correspondingly. Characteristic times are functions of all three emittances and particle number and have positive sign for a heating process and negative for cooling one. The negative sign of the lifetime corresponds to the particle loss and the sign of the lifetime can be positive in the presence of particle injection, when particle number increases. Index j in Eq.(1) is the number of process involved into calculations. The algorithm structure is designed in such a way that allows including any process into calculation, which can be described by cooling or heating rates. Numerical solution of the system (1) is performed using Euler method with automatic step variation. Result of the simulation is the emittance and particle number time dependencies. The ion ring optic

structure is necessary only for intrabeam scattering (IBS) simulation. The IBS growth rates are calculated in accordance with one of analytical models using ring lattice functions imported from output file of the MAD program.

Step of the system (1) integration over time is determined by characteristic times of investigated effects and calculation speed can be very fast. However, in some cases the basic physical model can not provide realistic simulation mainly due to basic assumption about Gaussian shape of the ion distribution function. This assumption is more or less realistic in an equilibrium state of the ion beam when the equilibrium is determined by many processes of stochastic nature. If the equilibrium does not exist due to fast particle loss or at initial stage of the beam cooling the ion distribution function can be far from Gaussian. The same situation takes place in an experiment with internal targets which dimensions are not coinciding to the ion beam dimensions. The ionization energy losses of the ion beam in the target can not be correctly calculated in the frame of this model also.

Investigation of the ion beam dynamics at arbitrary shape of the distribution function is performed using multi particle simulation in the frame of Model Beam algorithm. In this algorithm the ion beam is presented by array of modeling particles. The heating and cooling processes involved into simulations lead to change of the particle momentum components and particle number, what is calculated in accordance with step of dynamics simulation over time. Each effect is located in some position of the ring characterizing by the ring lattice functions. Transformation of the beam inside the ring is provided using linear matrix at random phase advance between the effect locations. Results of the simulations can be presented both as the beam profile evolution in time or as time dependencies of the beam emittance and particle number.

The real ion ring optic structure is necessary only for IBS diffusion power calculation only. The change of the particle momentum due to IBS is calculated on the basis of one of the analytical models as in the case of RMS dynamics simulation.

For simulation of the IBS process through Coulomb interaction between ions the Tracking algorithm is used. One of the goals of this algorithm development is to simulate a formation of crystalline state of the ion beam. In the crystalline state of the ion beam the IBS process can not be treated in the frames of analytical models, which are based on assumption of Gaussian shape of the ion distribution function. To speed up the calculations in the tracking algorithm the IBS simulation are performed using Molecular Dynamics technique presuming periodical ion distribution in the longitudinal direction. Therefore, this algorithm can be used for coasting beam only.

In the frame of the tracking algorithm the particle motion equations are integrated in the real optic structure of the ring. The ring structure is imported from input MAD file. Each cooling or heating effect involved into calculations together with IBS is located in some optic element. Calculation of the particle co-ordinates variation due to action of an effect is provided using the effect MAP. The effect position in the ring is described in input MAD file using special marks.

Structure of basic objects of the BETACOOL namely - the models of the ion ring and the ion beam, are developed in such a way, that allows realizing all three algorithms at the same input parameters. The heating and cooling effects are realized on the basis of common standard and at the same parameters can be used in each algorithm.

STRUCTURE OF EFFECTS

In the present version of the program the ion beam dynamics can be simulated taking into account one or a several effects from the following list (Figure 1):

1. Electron cooling,
2. Scattering on residual gas,
3. Interaction with internal target,
4. Collisions with encounter beam in the collider mode of the ring operation,
5. Particle losses at cooling section, collision point, acceptance, etc.,
6. Intrabeam scattering,
7. Additional External heating of the ion beam (by artificial noise source, for instance).

Algorithms for beam-beam effect, stochastic and laser cooling are under development now.

The effect structure permits to uniformly use each effect in all basic algorithms. For this purpose each effect is presented by three models: transformation map, kick of the ion momentum, characteristic time calculation.

The effect used as a transformation map is associated with some optic element of the ring and its position is marked in input file. The map transforms the particle co-ordinates from the entrance to the exit of the element and calculates particle loss probability.

On the basis of transformation map in each effect the procedures for calculation of the particle momentum kick and for characteristic time calculation are developed. Calculation of the momentum kick is used in Model Beam algorithm, characteristic times are necessary for RMS dynamics simulation.

The most develop effects are electron cooling and intrabeam scattering which includes a lot of physical models and different numerical algorithms.

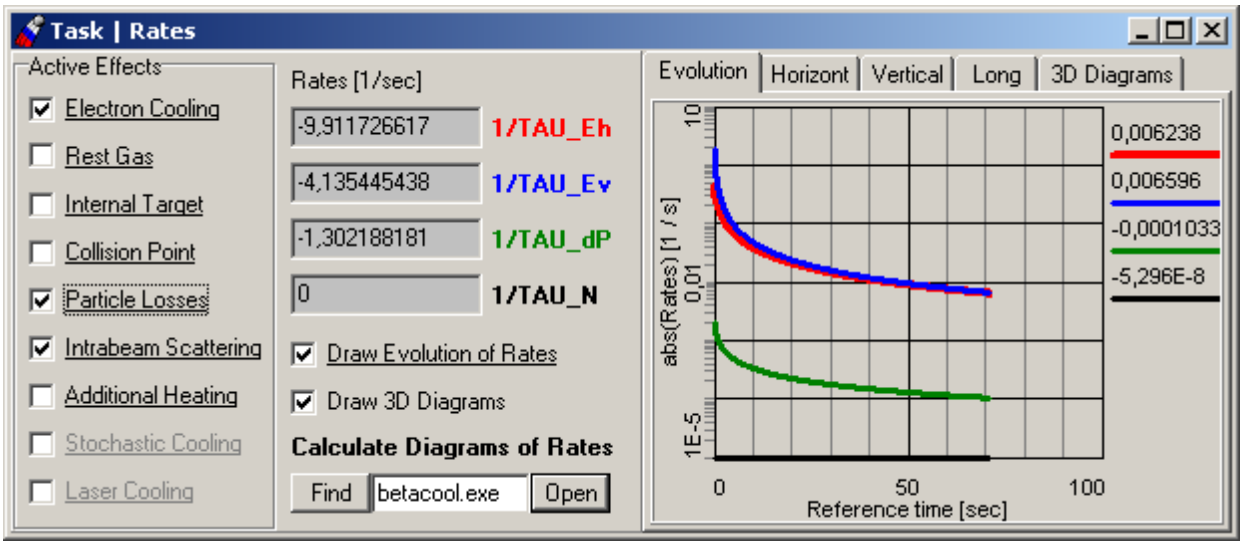

Figure 1: Form for the control of active effects and visualisation of growth rate evolution

ELECTRON COOLING

Structure of the effects can be illustrated on example of electron cooler model.

Usually an action of electron cooling on the ion dynamics inside a storage ring is described using a few standard simplifications:

1. Angular deviation of the longitudinal magnetic field line is substantially less than the ion beam angular spread.
2. Ion transverse displacement inside the cooling section is substantially less than electron beam radius.
3. Ion beam temperature is substantially larger than electron one and ion diffusion in the electron beam can be neglected.
4. Electron beam has a round shape of cross-section and uniform density distribution in the radial direction.

Under these assumptions and using asymptotic of the analytical friction force presentation the formulae for characteristic times of emittance and momentum spread decrease at electron cooling were obtained [9]. In the first version of the BETACOOL program electron cooling was simulated in accordance with this model [2]. This model is also used in a few program dedicated to electron cooling simulation. However this model can not cover all possible versions of the electron cooling system design.

In the last time modifications of the usual configuration of the electron cooling system were proposed. To avoid instability of the ion beam related with extremely large density of the cooled beam it was proposed to use so called “hollow” electron beam – the beam at small density in the central part. Extension of the electron cooling method in the region of electron energy of a few MeV related with an RF acceleration of the electrons. In this case one can expect Gaussian distribution of the electrons in radial plane and, if the electron bunch is shorter than the ion one, in longitudinal direction also. Calculation of the cooling times in this case requires modification both the electron beam model and the base physical model.

Other expected peculiarity of the medium energy cooling system is a big length of the cooling section – up to about 20 - 50 m. To obtain very high accuracy of the magnetic field is difficult technical task and cost of the cooling system will strongly depend on the required level of the accuracy. Therefore, before design of the cooling section solenoid, one needs to investigate influence of the magnetic field line curvature on the cooling process. All the effects can be taken into account by numerical solution of the ion motion equations in the cooling section.

To solve all the problems related with the cooling process simulation a hierarchy of objects was developed in the frame of the BETACOOL program. Structure of the electron cooler presentation permits to extract procedures of different levels and to include them into calculation of the cooling process in other programs. The cooling simulation is based on a friction force calculation in the particle rest frame. The friction force can be calculated in accordance with one of the analytical models from a library or using results of numerical calculations imported from external file. The next layer of the simulation is related with a cooler representation as a map, transforming particle coordinates from entrance to the exit of the cooling section and calculating the ion loss probability due to recombination with electrons. Calculation of the cooler map is based on a model of electron beam that provide transformation of the ion velocity to the frame related with the electron beam and takes into account real geometry of the cooler. Now in the BETACOOL three models of electron beam are available for simulations: uniform cylinder, Gaussian cylinder and Gaussian bunch. Model of the hollow beam will be realized in nearest future. The cooler model takes into account variation of the magnetic field in the cooling section. For this aim the co-ordinates of the electron beam trajectory inside cooling section is input from additional

file and the ion motion equations are solved numerically inside the cooler.

The map of the cooler can be used directly in the frame of the Molecular Dynamics algorithm, or in other tracking procedures. On the basis of the map one can calculate kick of the ion momentum after crossing the cooling section that is necessary for simulation of the ion distribution evolution in the frame of the Model Beam algorithm. The map of the cooler is used also for the cooling rate calculation that is necessary for RMS dynamics simulation. The cooling rate calculation can be performed using two model of the ion beam – the cooling rates for "RMS particle", or cooling rates for the ion beam with Gaussian distribution in all degrees of freedom.

INTRABEAM SCATTERING

Intrabeam scattering (IBS) in the ion beam causes two processes: relaxation of the beam to thermal equilibrium between degrees of freedom and diffusion growth of 6D phase volume of the beam due to variation of lattice parameters along ring circumference.

All usually used numerical algorithms of IBS growth rate calculation are based on the model of the collisions proposed by A. Piwinski.

Three models for IBS calculation – Piwinski, Martini (extended Piwinski) and Jie Wei models are realized in new version of BETACOOL program for Gaussian distribution of ions over velocity. The Martini model does not require additional assumptions for calculation of the beam emittance growth times. Piwinski model can be deduced from Martini model neglecting a variation of dispersion and beta function along the ring orbit (uniform optics). In the model proposed by Jie Wei characteristic times of emittance variation are calculated for real lattice parameters of the ring under a few additional assumptions, which correspond to storage rings at ion energy over the transition energy (for instance RHIC, BNL).

Simulation of the intrabeam scattering (IBS) process is based on calculation of the particle momentum variation due to coulomb interactions with other particles of the beam. In the BETACOOL the particle momentum variation can be calculated using analytical expressions for diffusion coefficients or, for coasting ion beam, using Molecular Dynamics (MD) technique. On the basis of both approaches each optic element of the ring is presented as a map for IBS process.

The map of the IBS process based on direct calculation of the ion coulomb interaction can be combined with the transformation map of optic element calculated from external focusing fields. In such a form the IBS process is simulated in the frame of tracking algorithm based on MD technique. This algorithm uses as input parameters the particle array presenting the ion beam and characteristics of external focusing fields.

The map of IBS process based on analytical theory calculates the growth rates for three degrees of freedom by numerical evaluation of integrals over ion distribution function assuming that the distribution function has Gaussian shape. The values of the growth rates are used for calculation of the individual particle momentum variation. The calculation of the growth rates requires as input parameters the RMS beam emittance and lattice functions of the ring in given optic element.

In the frame of RMS dynamics simulation the growth rates calculated in each optic element are averaged over the ring circumference. This procedure requires as input parameters the RMS beam emittances and description of the ring optic structure. The ring optic structure can be presented in both variants: as a specification of the optic elements or as dependencies of the lattice functions on the longitudinal co-ordinate along the ring circumference. In the first case the program initially provides tracking of Twiss parameters along the ring and transforms the ring model to the form of the lattice functions.

In the frame of a few models for the IBS growth rates calculation one needs mean parameters of the ring only. In this case detail description of the ring optic structure is not necessary and the IBS process can not be presented by a transformation map. Such models of the IBS can be used only in Model Beam algorithm and RMS dynamics simulation.

ORDERED BEAM SIMULATION

The idea of crystalline beam has received a large interest by now. The achievement of very low temperature in the beam rest frame opens new possibilities in accelerator physics. The increase of the luminosity in the collider and in experiments with targets is a very important asset for investigation of rare radioactive isotopes. The ordered state of circulating ion beams was observed at several storage rings [10].

The simulation of 1D crystalline beam with BETACOOL code was done [11]. The sudden reduction of momentum spread in the ESR experiment is described with this code. The simulation shows good agreement with the experimental results and also with IBS theory.

For the achievement of ordered ion beams with a large number of particles and with a realistic cooling force a special strategy of the cooling process can be elaborated. The longitudinal component of IBS heating has the break-up where heating rates have very small value in comparison with the theoretical prediction. If the initial parameters of ion beams can be chosen near the break-up that the ordered state for a large number of particle $N = 10^5$ can be achieved for real cooling system with electron beam current I_{ecool} = 5 A (Figure 2). During cooling process we may apply additional heating in the transverse direction, for example, heating by an RF-kicker. Initially, the momentum spread continues decrease and emittance will increase. When the beam parameters have to satisfy the condition $T_\perp >> T_\parallel$ we can switch off the additional heating and the ion beam will continue to cool down to the ordered state. The experimental verification of the new strategy for the achievement of an ordered ion beam

with large density can open new possibilities in the accelerator physics.

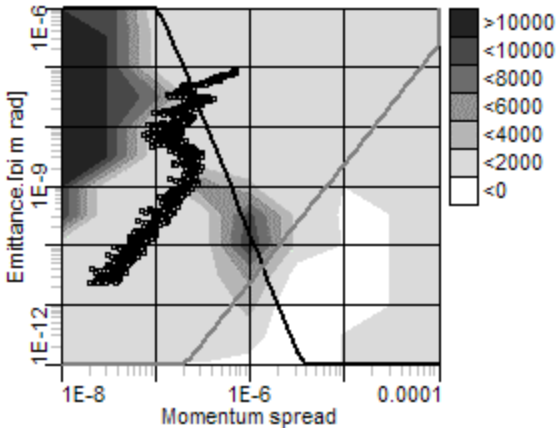

Figure 2: Numerical simulation for TARN-II with using of Molecular Dynamics technique. Black solid line – criterion of ordered state [10], grey solid line – equilibrium between transverse and longitudinal temperatures, grey islands – heating growth rates from IBS, points – particle dynamics with electron cooling

SOFTWARE STRUCTURE

The BETACOOL program is a part of software developed for electron cooling simulation. The software is divided in two independent parts: physical code, which is written using only standard C++ syntax and interface part, which is an executable program working under Windows environment. Connection between two parts of the program is provided using three types of the files: input, output and files used for control of the calculation process. Such a structure on the one hand allows to use the program on PC, to control the calculation process and analyze results during simulations. From the other hand the physical part of the program can be compiled for UNIX operation system and used for calculations independently on interface. The interface part in this case can be used for preparation of the input file and result visualization after completion of the calculations. All input and output files are in the text format and can be edited without interface program. The parameters in the input file are divided by groups in accordance with the structure of BETACOOL objects.

The interface part of the software consists of executable file Bolide.exe, *.dfm files containing information about BETACOOL exterior and input files for post processing of the calculated data. Development of the BETACOOL exterior is possible without recompilation of the Bolide.exe file. The interface part provides also service in work with the file structure on the disc.

The physical part of the software consists of the executable file Betacool.exe compiled for Windows or UNIX operation system and files of input parameters. For intrabeam scattering calculation one needs to use file of lattice parameters, for instance, MAD file. Additional input files are used for electron cooling simulation at friction force calculated by other program and for input of magnetic field errors in the cooling section.

The source code of the physical part of the software consists of tree relatively separated parts:

- interface part, which supports the format of input and output files common with the BOLIDE system,
- library of base numerical algorithms including description of dimensional variables, templates of the program self counters, procedures for matrix algebra, algorithms of numerical solution of differential equations,
- physical codes describing objects of the program and procedures with them.

Structure of the BETACOOL program exterior corresponds to the structure of general objects in the source code and correspondingly to the structure of input file.

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