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Simulation of HESR experiments with internal target using BETACOOL program

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Abstract

Numerical simulations of experiments with internal target for the new storage ring HESR (GSI, Darmstadt, Germany) are presented. BEATCOOL program for simulation of the ion beam parameters in the approximation of a Gaussian beam was used. Molecular dynamics technique was applied for the simulation of the intrabeam scattering. The possibility of using the MD method for simulation of a high density ion beam in experiments with an internal target is discussed.

1.Parameters of simulation

The injection energy for HESR is varied in the interval from 0.8 GeV to 15 GeV. Initial emittances and momentum spread are inversely dependent on the relativistic factor γ of the ion beam. Main parameters of simulation for HESR are presented in Table 1 and Table 2.

Table 1.Parameters of the HESR and initial beam parameters.

Circumference, C, m	449.467			
Type of ion	Antiproton			
Injection energy, E_{pbar}, GeV	0.8	3	7	14
Relativistic factor, γ	1.85	4.22	8.51	16.03
Initial transversal emittance, $\varepsilon_\perp$,m·rad	1×10^{-6}	4×10^{-7}	2×10^{-7}	1×10^{-7}
Initial momentum spread, $\Delta p/p$	5×10^{-4}	2×10^{-4}	1×10^{-4}	5×10^{-5}
Number of particles, N_i	5×10^{10}			
Gamma-transition, γ_{tr}	19.72			
Tune, Q_x Q_y	10.67 8.17			

Table 2.Parameters of the electron cooling system.

Electron current, I_e, A	0.1, 1
Electron beam radius, r_e, cm	1
Magnetic field strength, B, kGs	5
Length of cooling section, L, m	30
Beta functions in cooling section: β_{hor}, m β_{ver}, m	100 100
Longitudinal electron temperature, $T_\parallel$, meV	0.2
Transversal electron temperature, $T_\perp$, meV	100

2.Jet target

The numerical simulation of the cooling process in experiments with the jet target are produced. The main parameters of the jet target are presented in Table.3.

Table 3. Parameters of jet target.

Atomic mass, A	1
Charge, Z	1
Density, cm^{-2}	5×10^{15}
Beta functions in target section: β_{hor}, m	1.5
β_{ver}, m	1.5

BETACOOL program uses the Gaussian distribution of beam parameters and simulates the heating or cooling time rates from different effects which are included in the simulation: electron cooling (ECOOL, Derbenev-Skrinsky model), intrabeam scattering (IBS, Piwinski model) and internal jet target (JET, Bethe-Bloch equation).

Only with uniform target thickness in radial direction inside the beam radius the distribution function will remain a Gaussian one after crossing the target. This condition is well satisfied for a gas storage cell or a solid target, which overlaps the total cross section of the beam. In this case the parameters of the circulating ion beam vary at a single pass through the target in accordance with the following expression:

$$\begin{cases} \Delta\varepsilon_{h,v} = \dfrac{\beta_{h,v}\theta_{str}^2}{2} + \left(\dfrac{(1+\alpha_{h,v})^2}{\beta_{h,v}} D_{h,v}^2 + 2\alpha_{h,v} D_{h,v} D'_{h,v} + \beta_{h,v} {D'_{h,v}}^2 \right) \times \left(\dfrac{\Delta p}{p} \right)_{loss}^2 \\ \Delta\varepsilon_{long} = \left(\dfrac{\Delta p}{p} \right)_{target}^2 \end{cases}, \qquad (1)$$

where $\beta_{h,v}$, $\alpha_{h,v}$ are horizontal and vertical beta and alpha functions, $D_{h,v}$, $D'_{h,v}$ are dispersions and derivatives of the dispersion in the target point.

When circulating in a storage ring an ion beam crosses the target at each turn the mean energy loss leads to deceleration of the beam

$$\Delta E_{loss} = 2\xi \left(\ln \frac{E_{\max}}{I} - \beta^2 \right), \qquad (2)$$

and energy losses for momentum spread can be written as

$$\left(\frac{\Delta p}{p} \right)_{loss}^2 = \left(\frac{\gamma}{\gamma+1} \right)^2 \frac{\Delta E_{loss}^2}{E^2}. \qquad (3)$$

The standard deviation of the distribution function for a thick target is given by:

$$E_{str}^2 = \xi E_{\max} \left(1 - \frac{\beta^2}{2} \right), \qquad (4)$$

The fluctuations of the energy loss lead to an the growth of the ion beam momentum spread after a single crossing of the target in accordance with:

$$\left(\frac{\Delta p}{p} \right)_{target}^2 = \left(\frac{\gamma}{\gamma+1} \right)^2 \frac{E_{str}^2}{E^2}. \qquad (5)$$

Fig.1 shows the positive values of the heating rates for IBS and JET, the absolute values of cooling rates for ECOOL. Due to the coupling between the transverse and longitudinal degree of freedom the IBS heating rates have negative values in the range of small emittances and small momentum spread for horizontal components, and in the range of small emittances and large momentum spread for longitudinal component. In this simulation we assume horizontal and vertical emittances are equal. The vertical components of time rate do not differ much from horizontal ones.

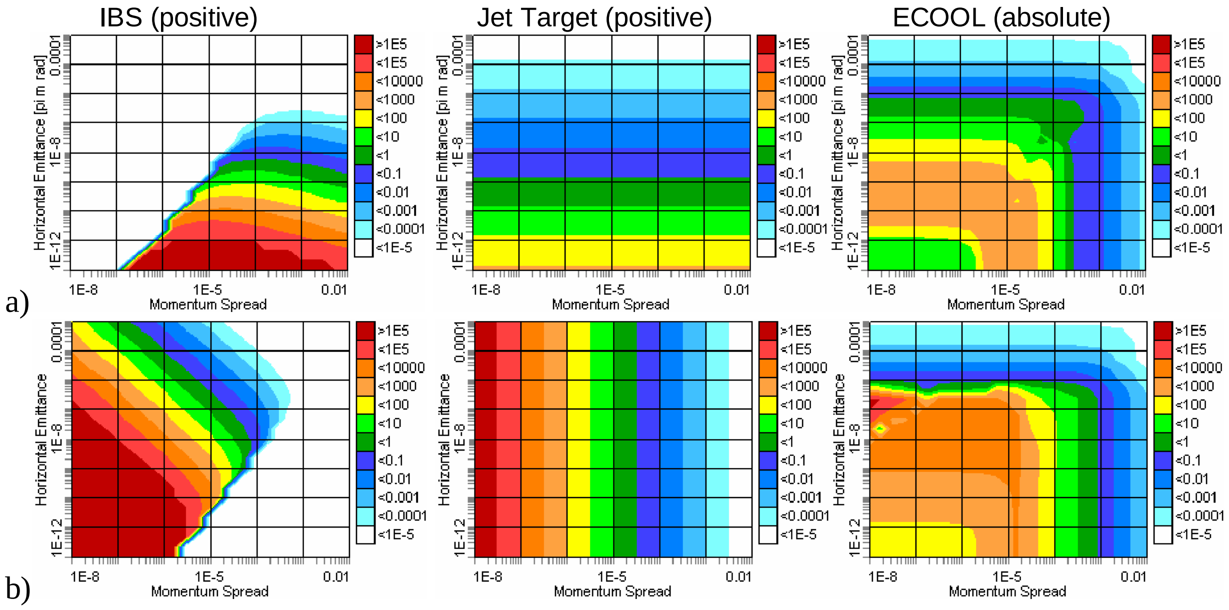


Fig.1. The dependence of time rates components on emittances and momentum spread.
a) horizontal, b) longitudinal.
E_{pbar} =3 GeV, N_i = 5×10^{10}, r_e = 1 cm, I_e = 1 A.

The summary of IBS and ECOOL time rates are presented in Fig.2. The electron cooling system has more powerful friction force in longitudinal direction than in transverse one. The heating rates for the internal jet target have linear dependencies for the transverse component on the emittance and for the longitudinal component on the momentum spread (Fig.1). The summary of IBS, ECOOL and JET heating rates shows that the jet target is effective only for large emittance and large momentum spread (Fig.3). It means that the equilibrium point for ion beam parameters is mainly defined by IBS and ECOOL.

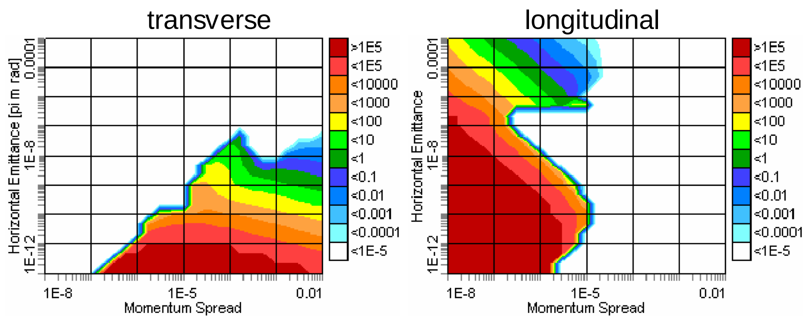


Fig.2. The dependence of transverse and longitudinal components of heating rates on emittances and momentum spread.
IBS +ECOOL. E_{pbar}=3 GeV, N_i = 5×10^{10}, r_e = 1 cm, I_e = 1 A.

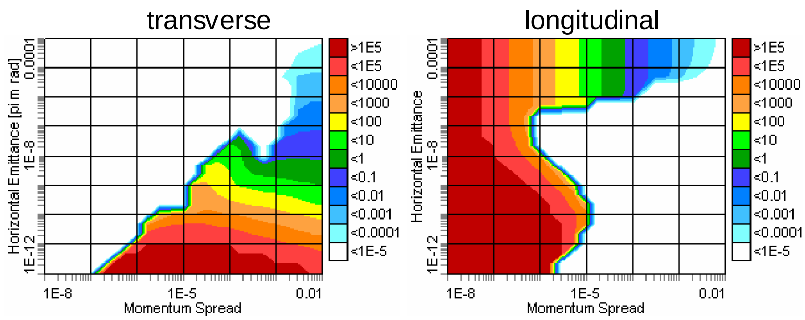


Fig.3. The dependence of transverse and longitudinal components
of heating rates on emittances and momentum spread.
IBS+ECOOL+JET. E_{pbar}=3 GeV, N_i = 5×10^{10}, r_e = 1 cm, I_e = 1 A.

Using 3-D diagrams of space phase (Fig.2,3) we can easily estimate when the equilibrium between heating and cooling effects can be achieved. If all three components of 3-D diagrams have the overlapped region with white color it means that the equilibrium can be found. If no white overlapped regions – no equilibrium and ion beam emittances and momentum spread will increase for any initial parameters.

For simulation of the cooling time the evolution dynamics of ion beam parameters is used (Fig.4). The initial parameters of the ion beam are chosen in accordance with injection energy (Table.1). The comparison of 3-D diagrams to the dynamics simulation shows that initially the ion beam is cooled very fast in longitudinal direction when the longitudinal component of IBS is compensated by the longitudinal cooling force. After that the cooling only in the transverse directions is continued but momentum spread still increases. And finally the ion beam achieves the equilibrium point which can be found in the overlapping area of 3-D diagrams on Fig.3.

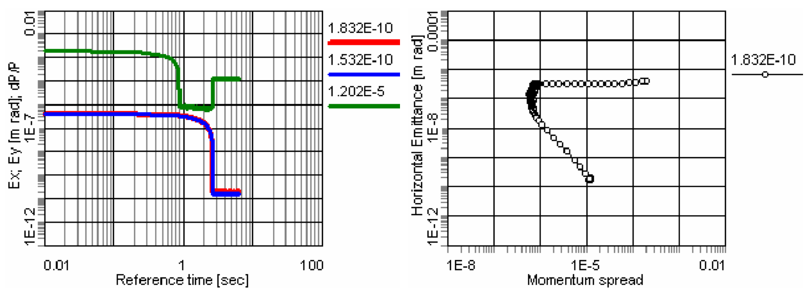
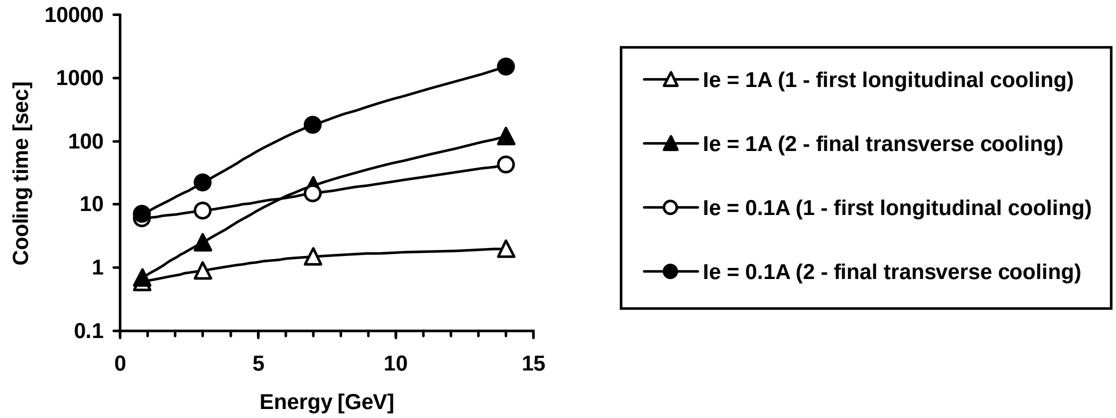


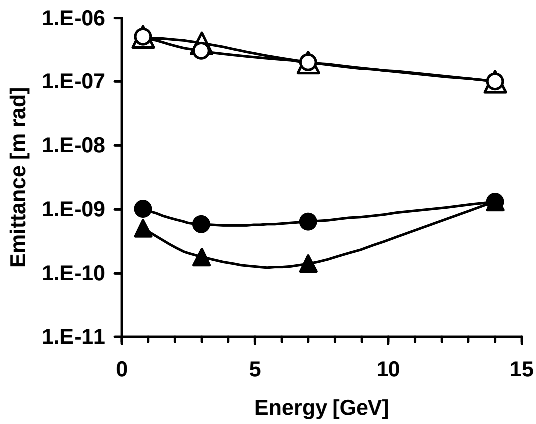
Fig.4.Dynamics of beam parameters. Green line corresponds to momentum spread,
red and blue lines – to the horizontal and vertical emittances.
IBS+ECOOL+JET. E_{pbar}=3 GeV, N_i = 5×10^{10}, r_e = 1 cm, I_e = 1 A.

Equilibrium parameters of ion beam do not have large dependence on the particle energy (Fig.5b,c). But the cooling time (Fig.5a) strongly depends on the particle energy and changes by 500 times from 0.8 GeV to 14 GeV. The ratio between the final cooling time and the initial fast longitudinal cooling increases with the energy.

The cooling time has the linear dependence on the electron current (Fig.5a). Equilibrium emittances and momentum spread decrease only by 2-3 times when the electron beam current decrease from 1 A to 100 mA. In this case we can propose maximum available electron current in the start point of cooling process and decrease the electron current when the equilibrium point is achieved.



a)



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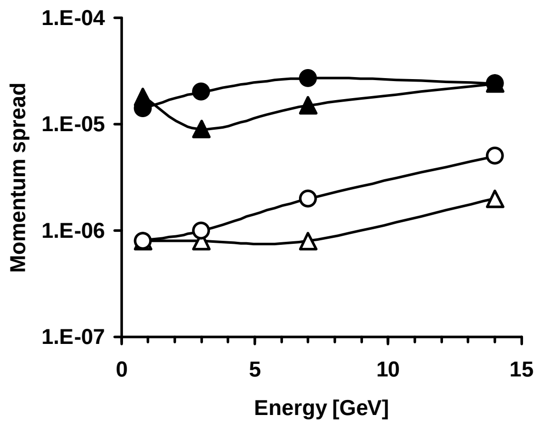


c)

Fig.5. The dependence of cooling time and ion parameters on particle energy
for the different electron beam current.
IBS+ECOOL+JET. $N_i = 5\times10^{10}$, $r_e = 1$ cm, $I_e = 0.1$ A, 1 A.

2. Pellet target

BETACOOL program has a Gaussian approximation for parameters of the ion beam and can not take the energy loss in the internal target. This approximation is good enough for simulation of experiments with jet target. An energy loss which particles have in the jet target is rather small in comparison to the growth of emittances and momentum spread. In this case we can assume that energy losses are compensated by the electron cooling and have not large influence on the cooling time and equilibrium parameters.

Ordinary distribution of particles is necessary for the simulation of the influence of energy losses (2) from the pellet target. In the existing model of BETACOOL program this task can not be resolved. Molecular dynamics (MD) technique was used in BETACOOL program for the simulation of crystalline beams. The same idea is proposed to the simulation of experiments with pellet target. The main problem of this method is a large calculation time for the simulation of the real density of ions in HESR. MD simulation can be used for the investigation of the influence of the pellet target on the cooling process and the particle distribution. The heating for each particle after passing through internal target is calculated in BETACOOL program as:

$$x_i' = x_i' + \sqrt{\frac{\theta_{str}^2}{2}} \times RandomGaussian ,$$
$$y_i' = y_i' + \sqrt{\frac{\theta_{str}^2}{2}} \times RandomGaussian , \qquad (6)$$
$$\left(\frac{\Delta p}{p}\right)_i^2 = \left(\frac{\Delta p}{p}\right)_i^2 + \left(\frac{\Delta p}{p}\right)_{t\arg et}^2 \times RandomGaussian - \left(\frac{\Delta p}{p}\right)_{loss}^2 .$$

For first checking of MD simulation for HESR experiments the number of particles was chosen 5×10^6 (10 particles per MD cell) that is by 4 order less than the real density. MD cell size can not be less than the ion beam size. In this case the length of MD cell was chosen about 1 mm. It is equal to the transverse beam size in the equilibrium point. 3-D diagrams of phase space and dynamics of beam parameters for particle number 5×10^6 without internal target are shown on Fig.6 and Fig.7.

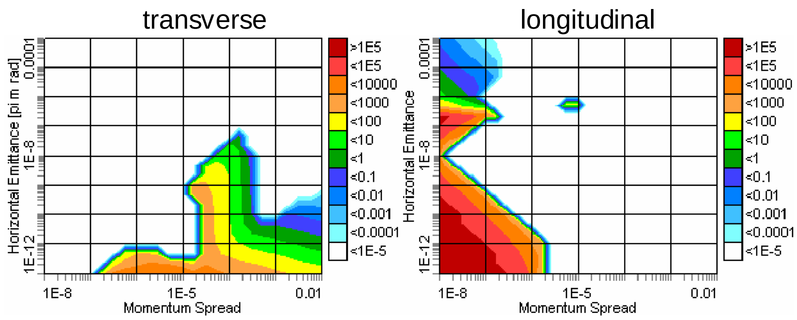


Fig.6. The dependence of transverse and longitudinal components of heating rates on emittances and momentum spread.
IBS+ECOOL. $E_{\rm pbar}$=3 GeV, $N_i = 5\times10^6$, r_e = 1 cm, I_e = 1 A.

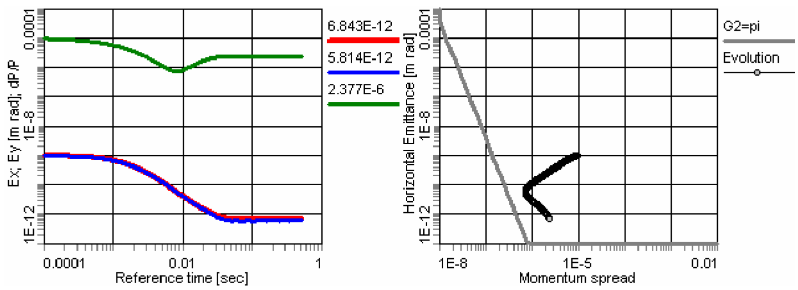


Fig.7. Dynamics of beam parameters. Green line corresponds to momentum spread, red and blue lines – to the horizontal and vertical emittances.
IBS+ECOOL. $E_{\rm pbar}$=3 GeV, $N_i = 5\times10^6$, r_e = 1 cm, I_e = 1 A.

The result of MD simulation for a few millions particles (10 particles per cell) shows that ion beam did not achieve the equilibrium between IBS and ECOOL. Emittances and momentum spread continuously decreased and achieved a very small value. BETACOOL program during the simulation with MD technique checks the number of particle crossing in the longitudinal direction. The cross number drops to zero value when parameters of ion beam had achieved the criterion of ordered state (solid gray line on 3-D diagrams Fig.8). The same result was obtained for MUSES Ion Ring (RIKEN, Japan) which was elaborated also for colliding experiments. To date we do not have a good explanation for this fact and can propose further investigations in this direction.

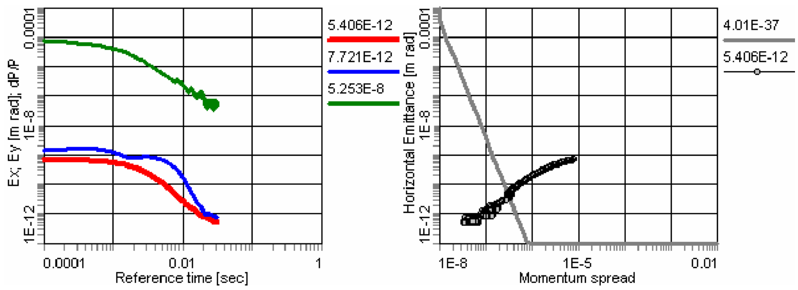


Fig.8. MD simulation (10 particles per cell). Green line corresponds to momentum spread, red and blue lines – to the horizontal and vertical emittances. IBS+ECOOL. E_{pbar}=3 GeV, $N_i = 5\times10^6$, r_e = 1 cm, I_e = 1 A.

The simple model of the pellet target was used in the simulation. The pellet target was approximated as frozen hydrogen cube with cross section 30×30 μm and thickness 20 μm. All pellets move in vertical direction with velocity 60 m/s with the same distance between them. Main parameters of the pellet target model are presented in Table.4.

Table 4. Parameters of pellet target model.

Atomic mass, A	1
Charge, Z	1
Density, g/cm^{-3}	0.0708
Density, atoms/cm^{-3}	4.26×10^{22}
Cross section, μm	30×30
Thickness, μm	20
Local density, atoms/cm^{-2}	8.53×10^{19}
Vertical velocity, m/s	60
Interval between pellets, mm	1
Effective density, atoms/cm^{-2}	8.92×10^{17}

The effective density is calculated from the local density of the pellet target, the cross section of pellets, distance between them and the transverse beam size in the target point. In the estimation of the effective density we assume that the cross section of pellets much more less than the beam cross section. The numerical integration of events when particles cross the pellet target gives the efficiency ~1% that is in a good agreement with the estimation of the effective density. For example, the efficiency for the jet target is 100%.

At initial point particles were generated with a Gaussian distribution. Initial emittances and momentum spread for MD simulation were chosen from 3-D diagrams of phase space (Fig.9) where the equilibrium point can be found from the overlapping of transverse and longitudinal components of heating rates and dynamics simulation (Fig.10). The parameters of MD simulation are presented in Table 5.

Table 5. Parameters of MD simulation.

Initial momentum spread	1×10^{-5}
Initial transverse emittance, $\varepsilon_\perp$,m·rad	1×10^{-9}
Beta functions in target section: β_{hor}, m β_{ver}, m	1.5 1.5
Particle energy, E_{pbar} GeV	3
Number of particles, N_i	5×10^7
Number of MD cell	5×10^5
MD cell size, mm	~1

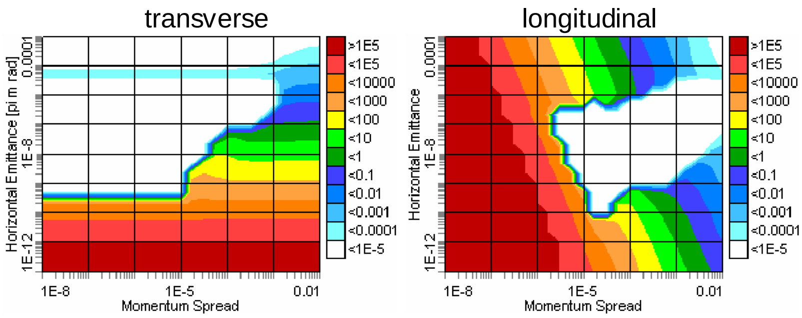


Fig.9. The dependence of transverse and longitudinal components
of heating rates on emittances and momentum spread.
IBS+ECOOL+PELLET. E_{pbar}=3 GeV, N_i = 5×10^7, r_e = 1 cm, I_e = 1 A.

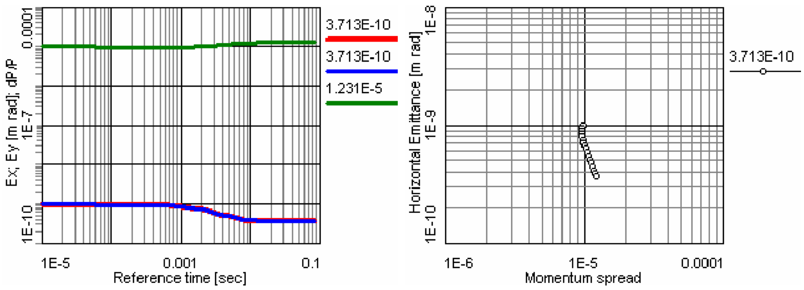
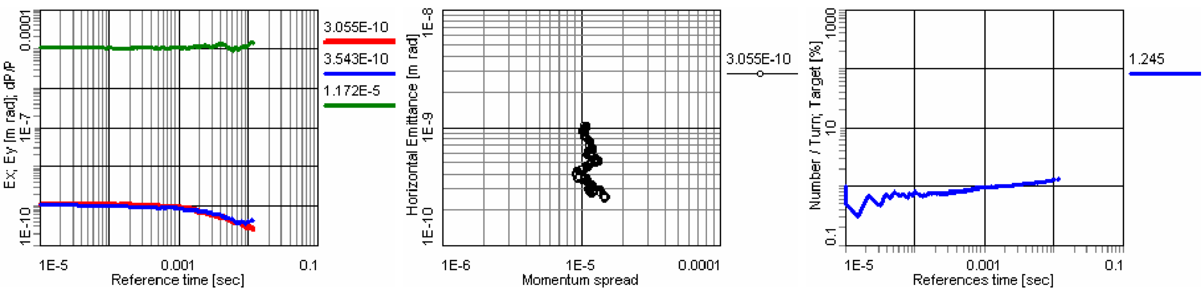


Fig.10. Dynamics of beam parameters. Green line corresponds to momentum spread,
red and blue lines – to the horizontal and vertical emittances.
IBS+ECOOL+PELLET. E_{pbar}=3 GeV, N_i = 5×10^7, r_e = 1 cm, I_e = 1 A.

For the chosen efficient density of the pellet target the equilibrium point are mainly defined by ECOOL and PELLET. The tracking simulation (Fig.11a,b) is in a good agreement with results of dynamics (Fig.10). The efficiency of pellet target slowly increases with time in accordance with the emittance decreasing (Fig.11c).



a) b) c)

Fig.11. MD simulation (100 particles per cell). a) evolution of beam parameters with time,
b) 3D diagram of phase space, c) efficiency of pellet target with time.
IBS+ECOOL+PELLET. E_{pbar}=3 GeV, N_i = 5×10^7, r_e = 1 cm, I_e = 1 A.

Results of MD simulations show that due to the energy loss in the pellet target a negative shift for the momentum spread distribution appears (Fig.12). It means that electron cooling system can not compensate the energy loss. For the investigation of the pellet target in the presence of intrabeam scattering we can choose the target effective density when heating rates from IBS and PELLET is equal.

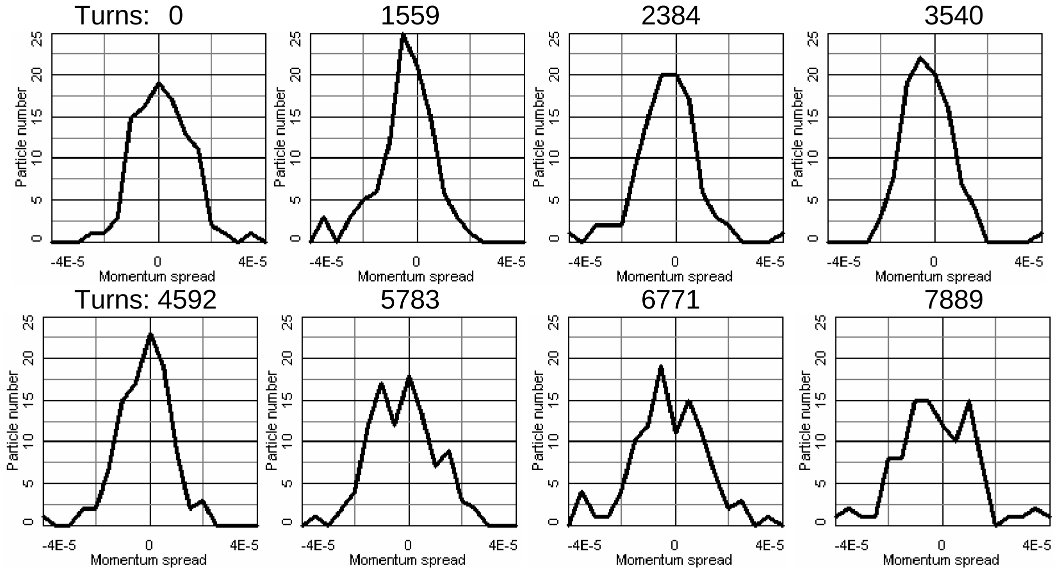


Fig.12. Evolution of momentum spread distribution with turn number.
IBS+ECOOL+PELLET. E_{pbar}=3 GeV, $N_i = 5\times10^7$, r_e = 1 cm, I_e = 1 A.

The main problem of MD simulation is large calculation time. The typical time of MD simulation is presented in Table 6. The simulation time is proportional to the square of particle number. From Table 6 we can estimate that for 1000 particle number per cell which are usually used for Monte Carlo method the simulation speed for given computer power is 1 turn per hour. In this case the MD simulation can be done for the total particle number in the ring $N_\text{i} = 5\times10^8$ which is still by two order less than proposed for HESR.

Table 6. Time of MD simulation.

Particles per cell	100
Integration step along trajectory, cm	~20
Computer power, MHz	800
Operation system	Windows NT.4
Speed of simulation, turns per hour	100

Conclusion

The simulation with BETACOOL program in the approximation of a Gaussian ion beam shows that for the chosen parameters of HESR experiments the equilibrium point is mainly defined by electron cooling and intrabeam scattering. The cooling time in this case is proportional to the electron beam current and inversely proportional to γ^2 which is predicted from the theoretical model of electron cooling.

The molecular dynamics technique is proposed for the simulation of the intrabeam scattering in the case of the ordinary distribution of ions. First test shows that MD simulation can estimate the energy losses due to scattering on the pellet internal target. Also MD simulation can be applied for the investigation of the hollow electron beam in the cooling system. Since the MD technique is time consuming the additional steps for decreasing of the calculation time can be offered.

The multiprocessor supercomputer can increase the speed of the simulation by few times. The new model of intrabeam scattering can be included in BETACOOL program when MD simulation is used one time per few thousand turns. Using these methods or combination of them will allow to reach in the MD simulation the necessary particle number which are proposed for HESR.