INTERNAL TARGET SIMULATION

1. Introduction

For simulations of the ion distribution function evolution due to action of different heating and cooling affects three basic algorithms are realized in BETACOOL:

- RMS dynamics simulation,
- Simulation of distribution function evolution using Monte-Carlo method (Model Beam algorithm),
- Multi particle tracking based on Molecular Dynamics technique.

To have a possibility to simulate internal target influence in the frame of all three algorithms this effect is presented in the program at three layers. For multi particle tracking the target is presented as a thin lens associated with some optic element of the storage ring and the target action on the ion is presented in the form of transformation map. On the basis of the map for RMS dynamics simulation a few models for characteristic time of emittance and particle number variation are developed. For investigation of long term processes in the frame of Model Beam algorithm the internal target is presented in the form related to kick of the ion momentum and loss probability calculation.

2. Map of the internal target

Map of an effect is used in turn by turn tracking procedure and has to provide variation of the particle coordinates in 6-dimensional phase space and calculate the particle loss probability. Internal target is treated in BETACOOL program as a thin lens, therefore the particle co-ordinates are not changed after crossing the target, but all three components of the particle momentum are changed and the particle can be loosed with some probability. Change of the transverse momentum components is related mainly with a multiple Coulomb scattering from nuclei of the target atoms. The Coulomb scattering distribution is well presented by the theory of Moliere. It is roughly Gaussian for small deflection angles, but at large angles (larger than a few r.m.s. value) it behaves like Rutherford scattering, having larger tail than a Gaussian distribution. Change of the longitudinal component of the particle momentum takes a place due to ionization energy losses. This process has also a statistic nature and in the simplest case can be described by two parameters: mean energy loss and standard deviation of the energy loss fluctuations.

Assuming Gaussian low of the processes one can calculate particle longitudinal and transverse momentum variations after single crossing the target by random generation of the scattering angle and energy loss:

$$\begin{aligned} x'_f &= x'_0 + \sqrt{\frac{\theta_{str}^2}{2}} \times \xi_1, \\ y'_f &= y'_0 + \sqrt{\frac{\theta_{str}^2}{2}} \times \xi_2, \\ \left(\frac{\Delta p}{p}\right)_f &= \left(\frac{\Delta p}{p}\right)_0 + \left(\frac{\Delta p}{p}\right)_{str} \times \xi_3 - \left(\frac{\Delta p}{p}\right)_{loss}, \end{aligned} \qquad (2.1)$$

where ξ_1, ξ_2, ξ_3 are independent random values having Gaussian distribution at unit standard deviation. For a thin target the rms scattering parameters θ_{str}^2, $\left(\frac{\Delta p}{p}\right)_{loss}$ and $\left(\frac{\Delta p}{p}\right)_{str}$ are linearly depend on the target density and can be calculated with the same analytical formulae in accordance with the local target density in the particle position. This model of the scattering process is realised now in BETACOOL program. More accurate model of a particle interaction with a target requires simulation of elementary processes inside the target. Algorithm for such a simulation is under development now.

Probability of the particle loss after crossing a target is determined mainly by three processes: single scattering on large angles, charge exchange and nuclear reactions in the target. Cross-section of the single scattering on large angle is calculated in accordance with Rutherford formula. The charge exchange in the target in the present version is taken into account only for fully stripped ions. For such an ion the program calculates cross-section of a capture of an electron in the target. For antiprotons particle losses due to charge exchange are not exist.

2.1. Calculation of the scattering rms parameters

Ions in matter lose energy primary because of ionization. At ion energy in an approximate range from 6 MeV to 6 GeV mean rate of energy loss (or stopping power) is given by the Bethe-Bloch equation:

$$\frac{dE}{d\rho x} = -KZ_P^2 \frac{Z_T}{A_T} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 E_{\max}}{I^2} - \beta^2 - \frac{\delta}{2} \right] \tag{2.2}$$

where E is the particle kinetic energy, ρ is the target density, x is co-ordinate along the ion trajectory, Z_P and Z_T are the charge number of projectile and target atoms, A_T is the target atomic number. E_{max} is the maximum energy loss in a head-on collision of the projectile with a target electron:

$$E_{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{M} + \left(\frac{m_e}{M} \right)^2}, \tag{2.3}$$

with m_e is the electron mass and M - the projectile mass. K is a constant determining by the following expression:

$$\frac{K}{A} = \frac{4\pi N_A r_e^2 m_e c^2}{A} = 0.307075 \text{ MeV}\cdot\text{g}^{-1}\cdot\text{cm}^2, \tag{2.4}$$

r_e is the electron classic radius, N_A is Avogadro number. I is the mean excitation energy, which is equal to 13.6 eV for hydrogen and $(10\pm1)\cdot Z$ eV for elements heavier than sulphur.

The density effect correction factor δ/2 in the equation (2.2) is much larger for a liquid or solid than for a gas and at very high energies

$$\frac{\delta}{2} \to \ln \frac{\hbar\omega_p}{I} + \ln \beta\gamma - \frac{1}{2}, \tag{2.5}$$

where the plasma energy $\hbar\omega_p$ is determined by electron density in the target:

$$\hbar\omega_p = \sqrt{4\pi n_e r_e^3}\, m_e c^2 / \alpha = 28.816 \sqrt{\rho_{[g/cm^3]} \left\langle \frac{Z}{A} \right\rangle} [eV] \tag{2.6}$$

Here α is the fine structure constant.

The energy loss determined by (2.2) is the mean value, for finite target thickness there are fluctuations of the energy loss. The ion distribution is skewed toward high values (Landau tail). For a thick layer (when $\Delta E >> E_{max}$) the distribution is nearly Gaussian.

Let us introduce the quantity ξ, which is proportional to the real density ρx of the target (ρ – target density in g/cm^3):

$$\xi = 0.1535 \left[\frac{MeV\, cm^2}{g} \right] \frac{Z_P^2}{\beta^2} \frac{Z_T}{A_T} \rho x. \tag{2.7}$$

Neglecting the effect density correction one can express the mean energy loss in the following form [1]:

$$\Delta E_{loss} = 2\xi\left(\ln\frac{E_{\max}}{I} - \beta^2\right), \tag{2.8}$$

and the decrease of the ion relative momentum can be written through the energy losses as

$$\left(\frac{\Delta p}{p}\right)^2_{loss} = \left(\frac{\gamma}{\gamma+1}\right)^2 \frac{\Delta E^2_{loss}}{E^2}. \tag{2.9}$$

The square of the standard deviation of the ion distribution function in the energy space for a thick target is given by:

$$E^2_{str} = \xi E_{\max}\left(1 - \frac{\beta^2}{2}\right), \tag{2.10}$$

When circulating in a storage ring an ion beam crosses the target at each turn the mean energy loss (2.8) leads to deceleration of the beam. For coasting beam without cooling the centre of the ion distribution in momentum space is displaced to smaller values. The fluctuations of the energy loss (2.10) lead to increase of the ion beam momentum spread after single crossing the target in accordance with:

$$\left(\frac{\Delta p}{p}\right)^2_{str} = \left(\frac{\gamma}{\gamma+1}\right)^2 \frac{E^2_{str}}{E^2}. \tag{2.11}$$

In the case of bunched ion beam the mean beam energy is stabilised by the action of RF field. Centre of the ion bunch crosses the RF cavity in the phase φ determining by the equation:

$$\Delta E_{loss} = eZV_{RF}\sin\varphi, \tag{2.12}$$

where V_{RF} is the amplitude of RF voltage.

In an experiment with internal target it is preferable to use coasting ion beam. In this case the mean energy loss is general factor limiting the experiment duration. At large enough target thickness, for instance for cluster or pellet target, one needs to apply electron or stochastic cooling to compensate this effect. This problem becomes more important in presence of dispersion in the target position. In this case the mean energy loss leads to displacement of the ion beam in horizontal plane and their fluctuations lead to increase of the ion beam horizontal emittance. When the mean energy loss is compensated, then equilibrium orbit displacement is negligible, but emittance growth has to be taken into account in beam parameter evolution calculations.

The compensation of the mean energy loss by longitudinal stochastic cooling or RF system leads to an ionisation cooling similar to muon cooling. The ion looses in the target the total momentum – transverse and longitudinal – but only longitudinal component loss is compensated. This leads to decrease of the ion angular deviation in respect to the equilibrium orbit. However, for ions this effect is negligible in comparison with the ion beam emittance growth due to multiple scattering through small angles in the target.

The r.m.s. scattering angle after crossing the target can be calculated using the formula:

$$\theta_{str}^2 = 2Nx\pi\left(\frac{Z_T Z_P r_p}{A_P \beta^2 \gamma}\right)^2 \left[\ln\left(\frac{\alpha_2^2}{\chi^2}\right) - 1 + \Delta b\right], \tag{2.13}$$

Here, Nx is the number of targets atoms per unit area, r_p – classical proton radius. The parametrs α_2, χ and Δb are given by the equations:

$$\alpha_2 = \frac{\text{a}}{\left(A_T^{1/3} + A_P^{1/3}\right) r_0}, \tag{2.14}$$

where $\text{a} = ¦ / p$ is De Broglie wavelength, A_T and A_P are the mass numbers of the target and the projectile, $r_0 = 1.3$ fm,

$$\chi^2 = 1.13\alpha_1^2 \left[1 + 3.33\left(\frac{Z_T Z_P}{137\beta}\right)^2\right], \tag{2.15}$$

where

$$\alpha_1 = \frac{\text{a}}{0.885 a_0 \left(Z_T^{2/3} + Z_P^{2/3}\right)^{1/2}}, \tag{2.16}$$

$a_0 = 0.529 \cdot 10^{-8}$ cm denotes the Bohr radius,

$$\Delta b = \frac{1}{Z_T}\left\{\ln\left[\frac{1130\beta^2}{Z_T^{4/3}(1-\beta)^2}\right] - u_{in} - \frac{\beta^2}{2}\right\}, \tag{2.17}$$

where u_{in} is a constant determined by the electron configuration of the target atom (from the Thomas-Fermi model one finds u_{in} = -5.8, for the H-atom exact calculation yields u_{in} = -3.6, for Li- and O-atoms the values of u_{in} are -4.6 and -5.0, respectively).

2.2. Calculation of the target geometry at the moment of the target position crossing

For a gas storage cell or for solid target overlapping the total beam the particle crosses the target each time when it crosses the target position in the ring. In the general case at each crossing the target position one needs to check is the particle really crosses the target or not. For this aim the map of the target for the particle momentum variation and loss probability calculation requires the current time of the target position crossing as an additional parameter.

More general case is presented by a pellet target geometry. Now the following model of the pellet target is used in the program (Fig. 2.1):

1. Pellets are approximated by cubes at uniform density, which is equal to the density of the frozen gas. To keep the real density of the pellet the cube dimensions have to be recalculated from the pellet diameter as $x_p = y_p = d\sqrt[3]{\pi/6}$, where d is the pellet diameter.

2. All the pellets move in vertical direction with the same velocity v_{pellet} and interval between them is equal to h.

3. The pellets move along the straight line shifted in horizontal direction by the distance of s relatively to the ion beam equilibrium orbit.

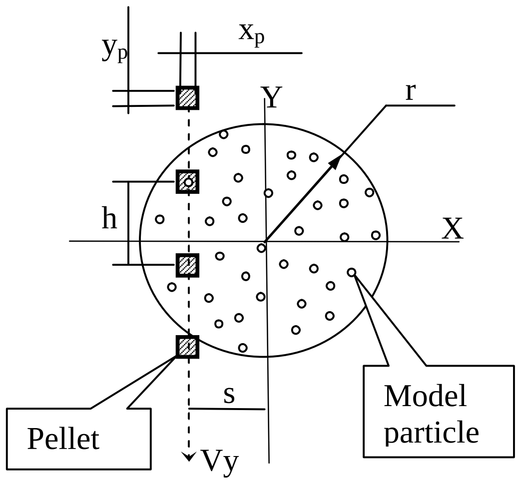
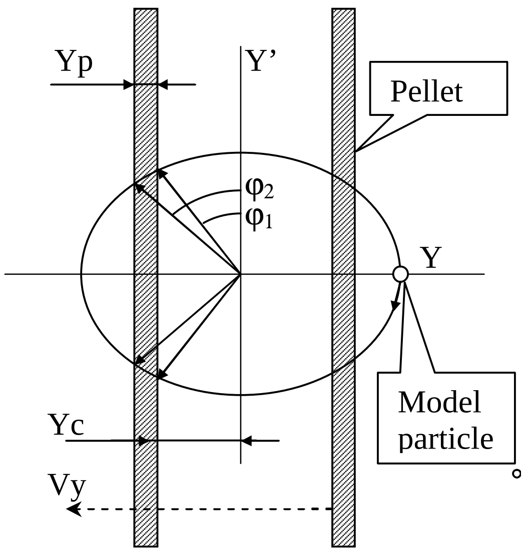


Fig. 2.1. Model of pellet target. x_p- horizontal size, y_p – vertical size, h – interval between pellets, s – horizontal shift, r – beam radius.



The pellet position at given moment of time is calculated under assumption that at zero time its vertical position is in median plane. The particle crosses the pellet when the following conditions are satisfied together:

$$|x_i - s| \leq \frac{x_p}{2} \tag{2.18}$$

$$\left| y_i + v_{pellet} t \pm h \left[\frac{y_i + v_{pellet} t}{h} \right] \right| \le \frac{y_p}{2} \tag{2.19}$$

where x_i and y_i are the horizontal and vertical ion co-ordinates at time t correspondingly, the brackets [] denote fractional part of the value.

This model can be used for simulation of cluster beam or gas jet target when the jet width is less than the ion beam diameter as well. In these cases the distance between the "pellets" has to be equal to the vertical dimension of the pellet $h = y_p$, and the target density has to be equal to effective cluster beam density or gas jet density with corresponding correction factor depending on geometry.

2.3. Calculation of the particle loss probability

The particle loss probability after crossing the target is determined by the following formula:

$$P_{loss} = \sigma_{total} Nx , \tag{2.20}$$

where Nx is the local target density in atoms per unit of square, σ_{total} is the sum of cross-sections of different reactions leading to the particle losses. Among them more important are the single scattering on large angles, capture of an electron in the target (in the general case – charge exchange), nuclear reactions in the target. Correspondingly in the program:

$$\sigma_{total} = \sigma_{ss} + \sigma_{ec} + \sigma_{nr} . \tag{2.21}$$

The cross-section for single scattering with an angle larger than the acceptance angle θ is [2]:

$$\sigma_{ss} = 4\pi \left(\frac{Z_T Z_p r_p}{A\gamma\beta^2} \right)^2 \frac{1}{\theta^2} , \tag{2.22}$$

where r_p is the proton classic radius. The acceptance angle, under assumption that the horizontal acceptance is substantially larger than the vertical one, is calculated as

$$\theta^2 = \frac{A_z}{\pi\beta_z} \left(1 - \left(\frac{\varepsilon_z}{A_z} \right)^2 \right), \tag{2.23}$$

where β_z is the beta function in the target position, A_z is the ring acceptance.

For completely stripped ions the cross-section of the electron capture inside the target can be calculated in accordance with the formula [3]:

$$\sigma_{ec} = 1.1 \cdot 10^{-8} [cm^2] \frac{\left(1 - \exp\left(-0.037 E_{eff}^{2.2}\right)\right)\left(1 - \exp\left(-2.44 \cdot 10^{-5} E_{eff}^{2.6}\right)\right)}{E_{eff}^{4.8}} , \tag{2.24}$$

where the effective energy in keV is determined as:

$$E_{eff} = \frac{KZ_T^{1.25}}{Z_P^{0.7}}, \tag{2.25}$$

K is the ion kinetic energy.

The cross-section of the nuclear reactions is estimated by the value:

$$\sigma_{nr} = \pi \cdot 10^{-26} [cm^2] \left(A_T^{1/3} + A_P^{1/3}\right)^2 / \beta^2 . \tag{2.26}$$

In the case of interaction with a pellet target the loss probability is calculated when the particle really crosses the pellet. In opposite case the probability is equal to zero.

3. Characteristic times of the beam parameter variation due to interaction with different types of target

The characteristic times of the beam parameter variation are used in the frame of RMS dynamics simulation. The physical model used in RMS dynamics simulation is based on the assumption that the ion beam has Gaussian distribution over all degrees of freedom and maximum of the energy spread coincides with the equilibrium energy. The parameters $(\Delta p/p)_{loss}$, $(\Delta p/p)_{str}$, θ_{str} are the functions of the target thickness x. Therefore only at uniform target thickness in radial direction inside the beam radius the distribution function will keep a Gaussian shape after crossing the target. This condition is well satisfied for gas storage cell or for solid target overlapping the total beam. However, even in this case the particle mean energy losses can not be treated correctly due to basic assumption of the model.

In the case of target with non-uniform density a self-consistent consideration of the beam parameter evolution in the frame of RMS dynamics algorithm is impossible. However for fast estimations of expected beam parameters one can use simplified model when an “equivalent” target at uniform density presents the real target. The density of this equivalent uniform target – “effective” density - has to be chosen to provide the same variation of the rms parameters of the particle distribution function as the real target. If the effective density is calculated during simulations trough the beam rms parameters in the first approximation such a model is self-consistent.

In the present version of the program, for simulation of the target at non-uniform density the model described in the chapter 2.2 is used. This model presumes that the target at uniform density overlap small part of the beam. In this case the target effective density can be calculated as a product of the real density and probability of the particle crossing the target. The probability of the crossing the target can be calculated analytically through geometry parameters of the target or using Monte Carlo method.

3.1. Gas storage cell and solid targets

For the target at uniform density the parameters $(\Delta p/p)_{loss}$, $(\Delta p/p)_{str}$, θ_{str} does not depends on the ion beam parameters and can be calculated using formulae (2.7 – 2.13). In this case the parameters of the circulating ion beam vary at single pass through the target in accordance with the following expression:

$$\begin{cases} \Delta\varepsilon_{h,v} = \dfrac{\beta_{h,v}\theta_{str}^2}{2} + \left(\dfrac{(1+\alpha_{h,v})^2}{\beta_{h,v}} D_{h,v}^2 + 2\alpha_{h,v} D_{h,v} D'_{h,v} + \beta_{h,v} D'^{\,2}_{h,v}\right) \times \left(\dfrac{\Delta p}{p}\right)_{loss}^2 \\ \Delta\varepsilon_{long} = \left(\dfrac{\Delta p}{p}\right)_{str}^2 \end{cases}, \qquad (3.1)$$

where $\beta_{h,v}$, $\alpha_{h,v}$ are horizontal and vertical beta and alpha functions, $D_{h,v}$, $D'_{h,v}$ are dispersions and derivatives of the dispersion in the target position. For the transverse rms emittances in the program the usual definition is used. The longitudinal beam emittance is determined as a square of the rms momentum spread. It is taken into account in the formula (3.1), that:

$$\theta_{rms}^{plane} = \frac{\theta_{rms}^{space}}{\sqrt{2}}. \qquad (3.2)$$

Then the emittance growth times can be calculated as follows:

$$\frac{1}{\tau_i} = \frac{1}{\varepsilon_i}\frac{\Delta\varepsilon_i}{T_{rev}}, \qquad (3.3)$$

where T_{rev} is the revolution period.

In the case of the target with uniform density the ion beam lifetime is calculated through the ion loss probability in accordance with the following formula:

$$\frac{1}{\tau_{life}} = -\frac{P_{loss}}{T_{rev}}. \qquad (3.4)$$

When the mean energy loss are sufficient in comparison with their fluctuations and does not compensated by additional cooling effect, in the program there is a possibility to include them into an "effective" increase of the momentum spread. In this case the longitudinal emittance variation after crossing the target is calculated as

$$\Delta\varepsilon_{long} = \left(\frac{\Delta p}{p}\right)_{str}^2 + \left(\frac{\Delta p}{p}\right)_{loss}^2. \qquad (3.5)$$

Of course, the treatment of the mean energy loss as an additional stochastic process is incorrect, but permits to estimate necessity of more correct simulations.

3.2. Gas jet, cluster beam and pellet targets

In the case when the target cross-section does not overlap the beam one or the target thickness does not uniform across the beam in the formulae (3.1) one needs to provide averaging over the target geometry. In the previous version of the BETACOOL program for this aim the probability of the target crossing by an individual particle *P(ε)* was introduced. At given probability value the emittance growth times are calculated as follows:

$$\frac{1}{\tau_i} = \frac{1}{\varepsilon_i}\frac{\Delta\varepsilon_i}{T_{rev}} P(\mathring{\varepsilon}), \qquad (3.6)$$

and $\Delta\varepsilon_i$ are calculated in accordance with (3.1) or (3.5) at mean target thickness. The ion beam lifetime is calculated through the ion loss probability by the same way as in formula (3.4):

$$\frac{1}{\tau_{life}} = -\frac{P_{loss}}{T_{rev}} P(\mathring{\varepsilon}). \qquad (3.7)$$

In the general case the crossing probability is a function of all three beam rms emittances and it can be calculated in accordance with a model of the target geometry or using Monte Carlo method.

3.2.1. Calculation of the target effective density from geometry parameters

The value *P(ε)* can be estimated as a ratio between beam cross section and the target one without any assumption about real geometry of the target position:

$$P(\varepsilon) = \begin{cases} 1 & if \quad S_b \le S_t \\ S_t / S_b & if \quad S_b > S_t \end{cases}, \tag{3.8}$$

where S_t is the effective target cross-section, $S_b = \pi r^2$ is the r.m.s. beam cross-section, $r^2 = \sigma_{hor}\sigma_{vert}$ – square of beam radius, σ_{vert} and $\sigma_{hor} = \sqrt{\sigma_{hor,bet}^2 + D^2\sigma_p^2}$ are the vertical and horizontal r.m.s. beam dimensions correspondingly. The betatron rms beam dimensions are calculated as usual: $\sigma = \sqrt{\varepsilon\left(1+\alpha^2\right)/\beta}$, and σ_p denotes the rms momentum spread. To take into account Gaussian distribution of the particles over transverse co-ordinates the beam cross-section has to multiplied by factor 2.

In accordance with the pellet model described in the chapter 2 (see Fig. 2.1) the effective cross-section of the pellet target is calculated by averaging the particle vertical position over time:

$$S_t = x_p 2\sqrt{r^2 - s^2}\,\frac{y_p}{h}. \tag{3.9}$$

The factor y_p/h appears after averaging over time assuming that the pellet has a constant vertical velocity. This formula can be used for simulation of cluster beam or gas jet target by the same way as described in the chapter 2.2.

The described model gives good estimation for the target influence on the r.m.s. parameters of the ion distribution function. However it ignores the fact that only a small part of the beam interacts with the target. For the transverse emittance growth it is not so important until the Courant-Snider invariant of the particle crossed the target is less than the total ion beam emittance. For the longitudinal degree of freedom the situation is more complicated. When the target cross-section is sufficiently less than the beam cross-section the center of the distribution function over momentum deviation does not change a position after crossing the target, because general part of the beam does not cross the target. The particles crossed the target form a tail of the distribution function into the region of small momentum. The formula (3.9) can give good estimation for intensity of the tail, but it is underestimate the tail length by the value of h/y_p. It can be very important for estimation of the electron cooling efficiency due to strong dependence of the cooling time on the ion velocity. This effect can be taken into account in the frame of Model Beam algorithm.

3.2.2. Monte Carlo method of the effective density calculation

More accurate self-consistent calculation of the target effective density is based on Monte Carlo method, which does not need any assumptions on the ion beam geometry. The method is based on the ion beam presentation in the target position in the form of the particle array, which has the same rms parameters as the total ion beam and matched with the ring lattice parameters in the target position. This model beam consists of the particles at the same charge and mass as in the real beam and differs from the real beam only by the particle number.

At the first step of the algorithm the model beam is generated in accordance with given rms parameters and lattice functions at the target position. After that the model beam is propagated through the target particle by particle. The algorithm uses the same model of the pellet target at density averaged over time as in the previous chapter. If the particle horizontal co-ordinate x_i satisfies the condition:

$$\left|x_i - s\right| \le \frac{x_p}{2} \tag{3.10}$$

the momentum components of this particle are changed in accordance with

$$\begin{aligned} x'_f &= x'_0 + \sqrt{\frac{\theta_{str}^2}{2}} \times \xi_1, \\ y'_f &= y'_0 + \sqrt{\frac{\theta_{str}^2}{2}} \times \xi_2, \\ \left(\frac{\Delta p}{p}\right)_f &= \left(\frac{\Delta p}{p}\right)_0 + \left(\frac{\Delta p}{p}\right)_{str} \times \xi_3 - \left(\frac{\Delta p}{p}\right)_{loss}, \end{aligned} \tag{3.11}$$

where ξ_1, ξ_2, ξ_3 are independent random values having Gaussian distribution at unit standard deviation. Here in difference with the internal target map calculation the rms scattering parameters θ_{str}^2, $\left(\frac{\Delta p}{p}\right)_{loss}$ and $\left(\frac{\Delta p}{p}\right)_{str}$ are calculated at effective target density, which is related with the real one as

$$\rho_{eff} = \rho \frac{y_p}{h}. \tag{3.12}$$

This effective density is used also for calculation of the particle loss probability.

If the particle is outside the target (condition (3.10) is not satisfied) its momentum components are not changed and the loss probability is equal to zero.

After propagation of all the particles through the target final rms parameters of the particle array are calculated. The characteristic times of the emittance variation are calculated than as follows

$$\frac{1}{\tau_i} = \frac{\varepsilon_{i,f} - \varepsilon_{i,0}}{\varepsilon_{i,0} T_{rev}}. \tag{3.13}$$

Here index *i* denotes one of degrees of freedom. The beam lifetime is calculated as

$$\frac{1}{\tau_{life}} = -\frac{\langle P_{loss} \rangle}{T_{rev}}, \tag{3.14}$$

where the brackets $\langle \ \rangle$ denote averaging over the particles.

Accuracy of the characteristic time calculation is determined by the particle number in the model beam. The particle number required to obtain the same accuracy is scaled with the beam and target parameters as σ_x / x_p. At the pellet diameter of 30 – 60 μm and the beam cross-section of a few millimetres the particle number has to be a few tens of thousands to obtain an accuracy of about a few percents. Accuracy of the calculations does not depend on the pellet velocity value and this algorithm can be applied without additional assumption. In the case when the pellet moves through the beam during many thousands of the beam revolution the accuracy of the algorithm can be improved by averaging on the particle betatron oscillations. Such a modification of Monte Carlo method is used in the frame of Model Beam algorithm and it is described in the next chapter.

4. Kick of the ion momentum due to interaction with target

4.1. Uniform density

In the frame of the basic physical model of the RMS dynamics simulation all the effects leading to deformation of the distribution function shape can not be taken into account. For simulation of the beam distribution function evolution in time the Mdel Beam algorithm is used in Betacool. In the frame f this algorithm the ion beam is presented as an array of model or test particles and all the effects changing the distribution function lead to variation of the particle momentum componets. Change of the particle momentum after one step of the motion equation integration is calculated in accordance with Langevin equation:

$$\Delta p / p = \Lambda \Delta T + \sqrt{D \Delta T}\xi \ , \tag{4.1}$$

where Λ is drift term, D is diffusion term, ΔT is step of the integration over time, ξ is Gaussian random number at unit dispersion. For interaction with internal target the diffusion and drift terms are calculated in respect to single crossing the target in accordance with formulae from chapter 2. The equation (4.1) can be rewritten in the following form:

$$\begin{aligned} x'_f - x'_0 &= \sqrt{\frac{\theta_{str}^2}{2} N_{cross}} \times \xi_1 , \\ y'_f - y'_0 &= \sqrt{\frac{\theta_{str}^2}{2} N_{cross}} \times \xi_2 , \\ \left(\frac{\Delta p}{p}\right)_f - \left(\frac{\Delta p}{p}\right)_0 &= \sqrt{\left(\frac{\Delta p}{p}\right)_{str}^2 N_{cross}} \times \xi_3 - \left(\frac{\Delta p}{p}\right)_{loss} N_{cross} , \end{aligned} \tag{4.2}$$

where N_{cross} is the number of the target crossing during interval ΔT, $\xi_{1,2,3}$ – independent Gaussian random numbers. When the target at uniform density fully overlap the beam the crossing number is simply equal to

$$N_{cross} = \frac{\Delta T}{T_{rev}} . \tag{4.3}$$

4.2. Number of the crossing in the case of pellet target

The number of the pellet crossing by a particle at given invariants of the motion during one step of integration over time ΔT can be calculated at the following additional assumption. Lets suppose, the pellet move across the beam slow enough. Such, that the pellet change its position in the beam during a few thousands of ion revolutions. In this case one can use simplified description of the particle betatron and synchrotron motion:

$$\begin{cases} x = \sqrt{I_x \beta_x}\cos\varphi_x + D\dfrac{\Delta p}{p} \\ y = \sqrt{I_y \beta_y}\cos\varphi_y \\ \dfrac{\Delta p}{p} = \sqrt{I_l}\cos\varphi_l \end{cases} \tag{4.4}$$

where $I_{x,y}$ – Courant-Snider invariants in horizontal and vertical plane, $\beta_{x,y}$ – corresponding beta functions in the target position. Invariant of the motion in longitudinal plane is equal to square of the amplitude of the particle momentum oscillations, in the case of coasting beam this invariant isequal to square of the particle momentum deviation and the momentum deviation keeps a constant value during revolution in the ring. φ_i are the phases of oscillation and at slow movement of the pellet they are can be treated as an independent random numbers uniformly distributed in the interval from 0 to $2\cdot\pi$.

Initially lets discuss situation with coasting ion beam. Lets at moment of t_0 the pellet is located in position where co-ordinates of its centre are x_C and y_C correspondingly. We use the same notation for the pellet parameters as in the Fig. 2.1, and $x_C = s$. The particle horizontal co-ordinate in the target position is inside the target when

$$\left| x - x_C \right| \le \frac{x_p}{2} \ , \tag{4.5}$$

that is realised when the phases of oscillations lies in the interval

$$\arccos\left(\frac{x_C - \dfrac{x_p}{2} - D\dfrac{\Delta p}{p}}{\sqrt{I_x \beta_x}} \right) \ge \varphi_x \ge \arccos\left(\frac{x_C + \dfrac{x_p}{2} - D\dfrac{\Delta p}{p}}{\sqrt{I_x \beta_x}} \right). \tag{4.6}$$

If the phase is uniformly distributed from zero to $2\cdot\pi$, the probability to have the particle horizontal co-ordinate inside the pellet P_x is equal to

$$P_x = \frac{\varphi_{x,2} - \varphi_{x,1}}{\pi}, \tag{4.7}$$

where $\varphi_{x,1,2} = \arccos\left(\frac{x_C \text{ « } \dfrac{x_p}{2} - D\dfrac{\Delta p}{p}}{\sqrt{I_x \beta_x}} \right)$. The probability to have a particle vertical co-ordinate inside the pellet is

$$P_y = \frac{\varphi_{y,2} - \varphi_{y,1}}{\pi}, \tag{4.8}$$

where $\varphi_{y,1,2} = \arccos\left(\frac{y_C \text{ « } \frac{y_p}{2}}{\sqrt{I_y \beta_y}}\right)$. Because of the pellet movement in vertical direction its co-ordinate is changed with time

$$y_C = y_0 - v_{pellet} t \,, \tag{4.9}$$

Assuming, that the vertical size of the ion beam is equal to 2*Y*, the probability to cross the target averaged over pellet movement can be estimated by the following expression:

$$P_{cross} = P_x \frac{v_{pellet}}{Y} \int_0^{Y/v_{pellet}} P_y(t) dt \,. \tag{4.10}$$

Using this value the number of the pellet crossing during period of ΔT can be calculated as follows

$$N_{cross} = \frac{\Delta T}{T_{rev}} P_{cross} \frac{2Y}{h} \,, \tag{4.11}$$

where multiplier $2Y/h$ gives a number of pellets crossing the beam at the same moment of time. Finally the number of the pellet crossings as a function of the particle Courant-Snider invariants can be expressed as:

$$N_{cross} = \frac{\Delta T}{T_{rev}} \frac{2 v_{pellet}}{\pi^2 h} \left(\arccos\left(\frac{x_C + x_p/2 - D\Delta p/p}{\sqrt{I_x \beta_x}} \right) - \arccos\left(\frac{x_C - x_p/2 - D\Delta p/p}{\sqrt{I_x \beta_x}} \right) \right) \times$$
$$\times \int_0^{T_{\max}} \left(\arccos\left(\frac{v_{pellet} t + y_p/2}{\sqrt{I_y \beta_y}} \right) - \arccos\left(\frac{v_{pellet} t + y_p/2}{\sqrt{I_y \beta_y}} \right) \right) dt \,, \tag{4.12}$$

where upper limit of the integration is given by

$$T_{\max} = \frac{\sqrt{I_y \beta_y} - y_p/2}{v_{pellet}} \,. \tag{4.13}$$

And, obviously, the number of crossings is zero if

$$\sqrt{I_x \beta_x} < x_C - x_p/2 - D\Delta p/p \,. \tag{4.14}$$

The case of bunched ion beam requires more detailed consideration due to low frequency of synchrotron oscillations in comparison with the frequency of betatron oscillations.

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